

Cristalli Fotonici

Corso di Elettromagnetismo Applicato
25 Novembre 2013

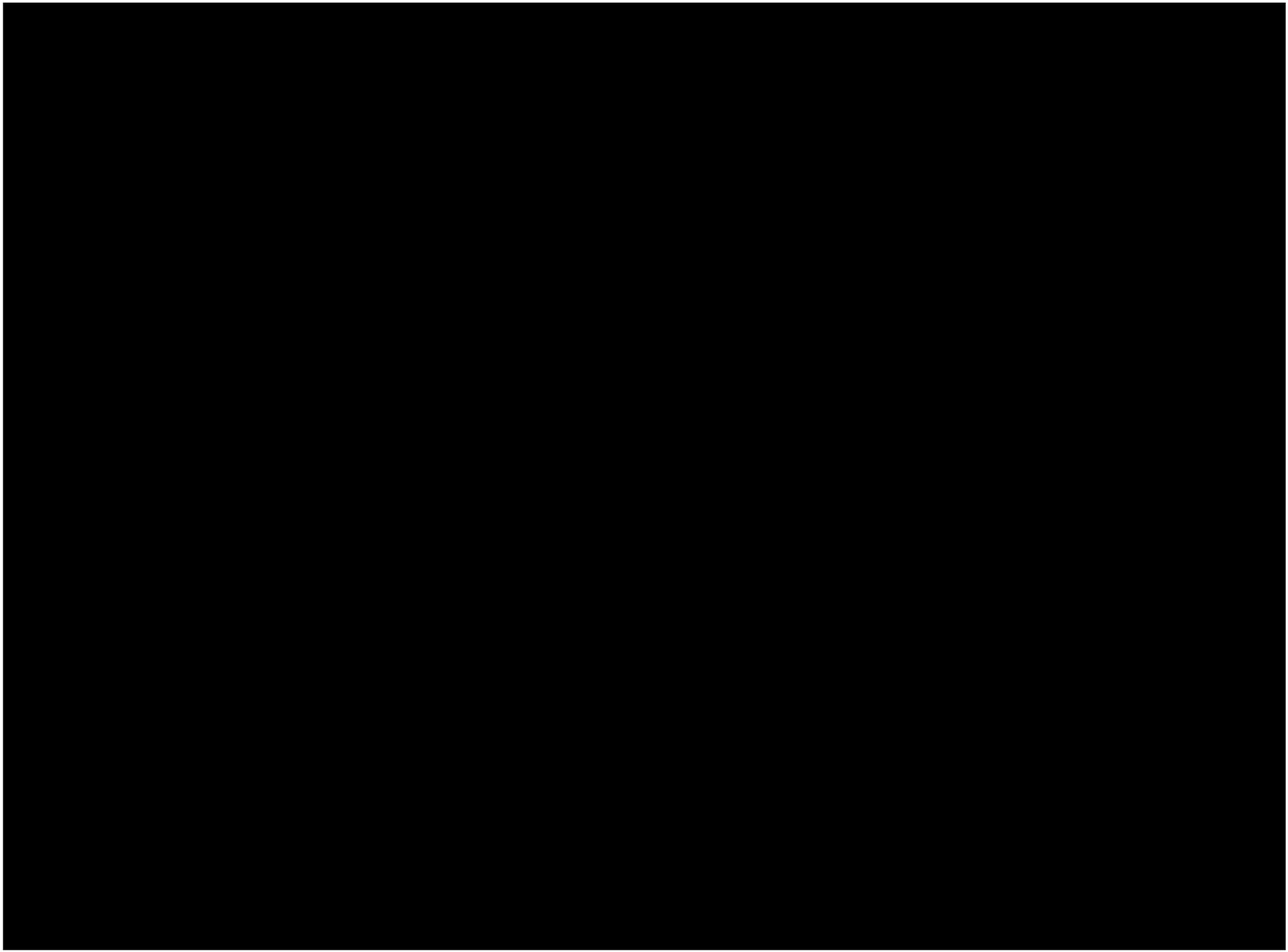
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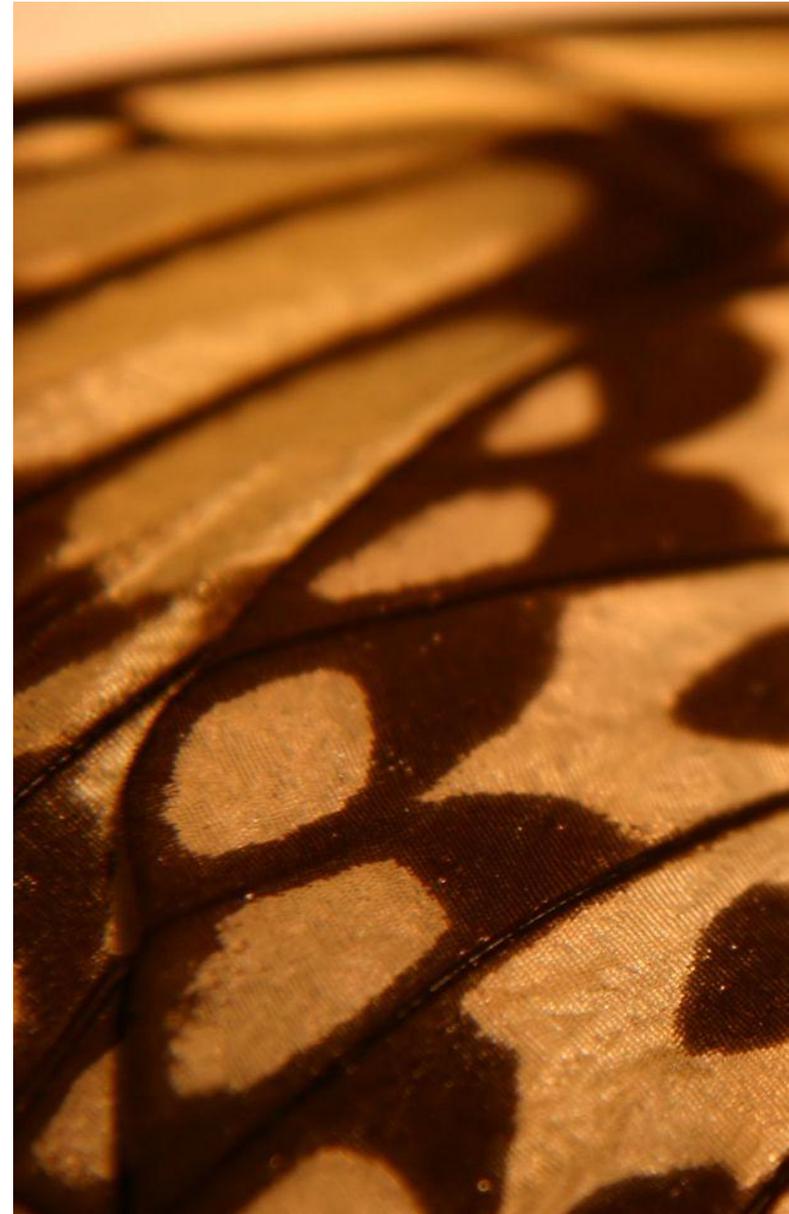
**UNIVERSITÀ DEGLI
STUDI DI PARMA**

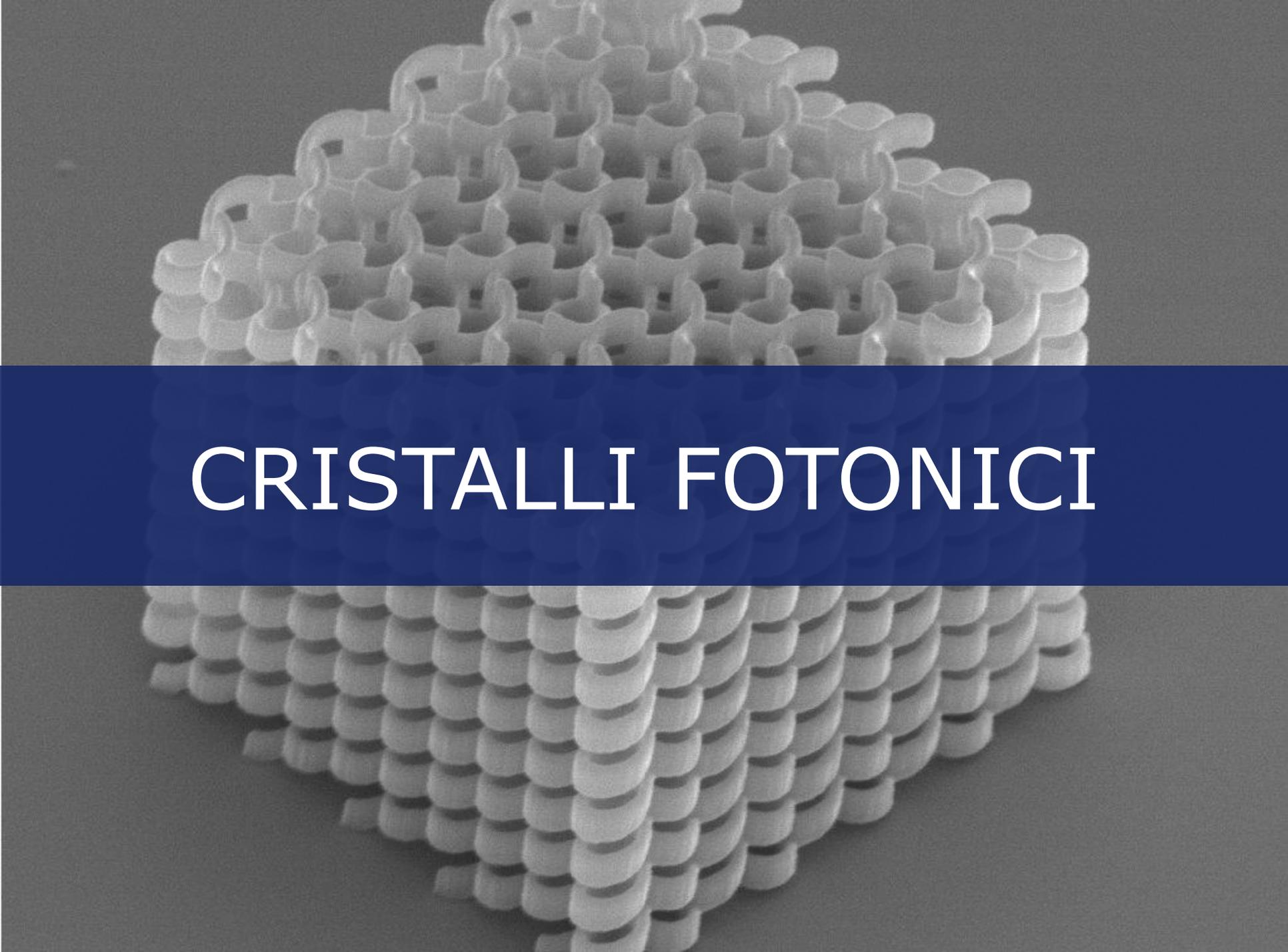


**DIPARTIMENTO
DI INGEGNERIA
DII DELL'INFORMAZIONE**



- Cristalli fotonici
- Photonic bandgap
- Fibre a cristallo fotonico
- Analisi e progetto di fibre a cristallo fotonico

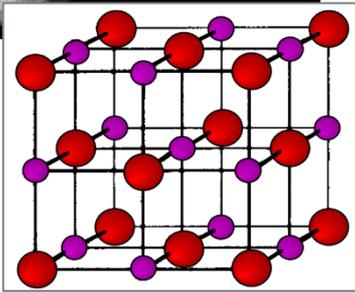




CRISTALLI FOTONICI



Salgemma
(NaCl)

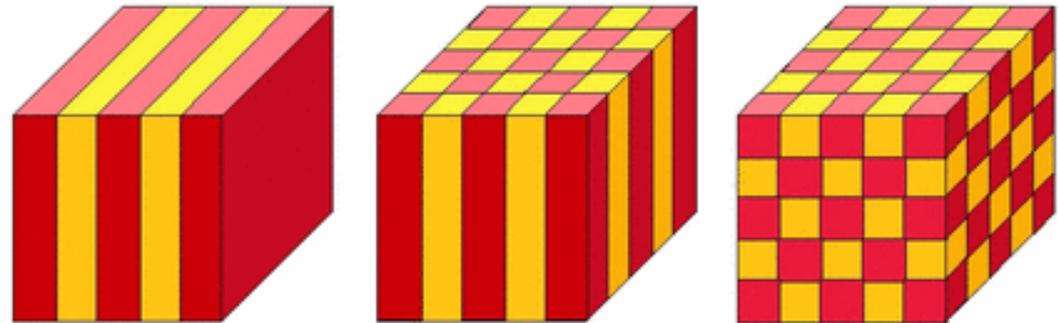


CRISTALLO:

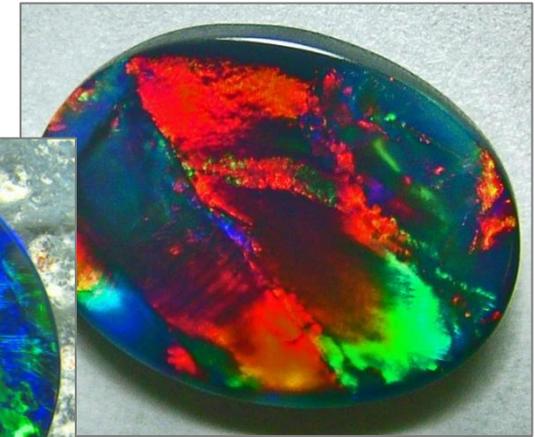
solido costituito da ioni, atomi o molecole disposti in maniera regolare, che si ripete periodicamente nello spazio

CRISTALLO FOTONICO:

struttura **dielettrica** con variazione **periodica** dell'indice di rifrazione, su scala paragonabile alla lunghezza d'onda della luce ($\sim 0.01 - 100 \mu\text{m}$)

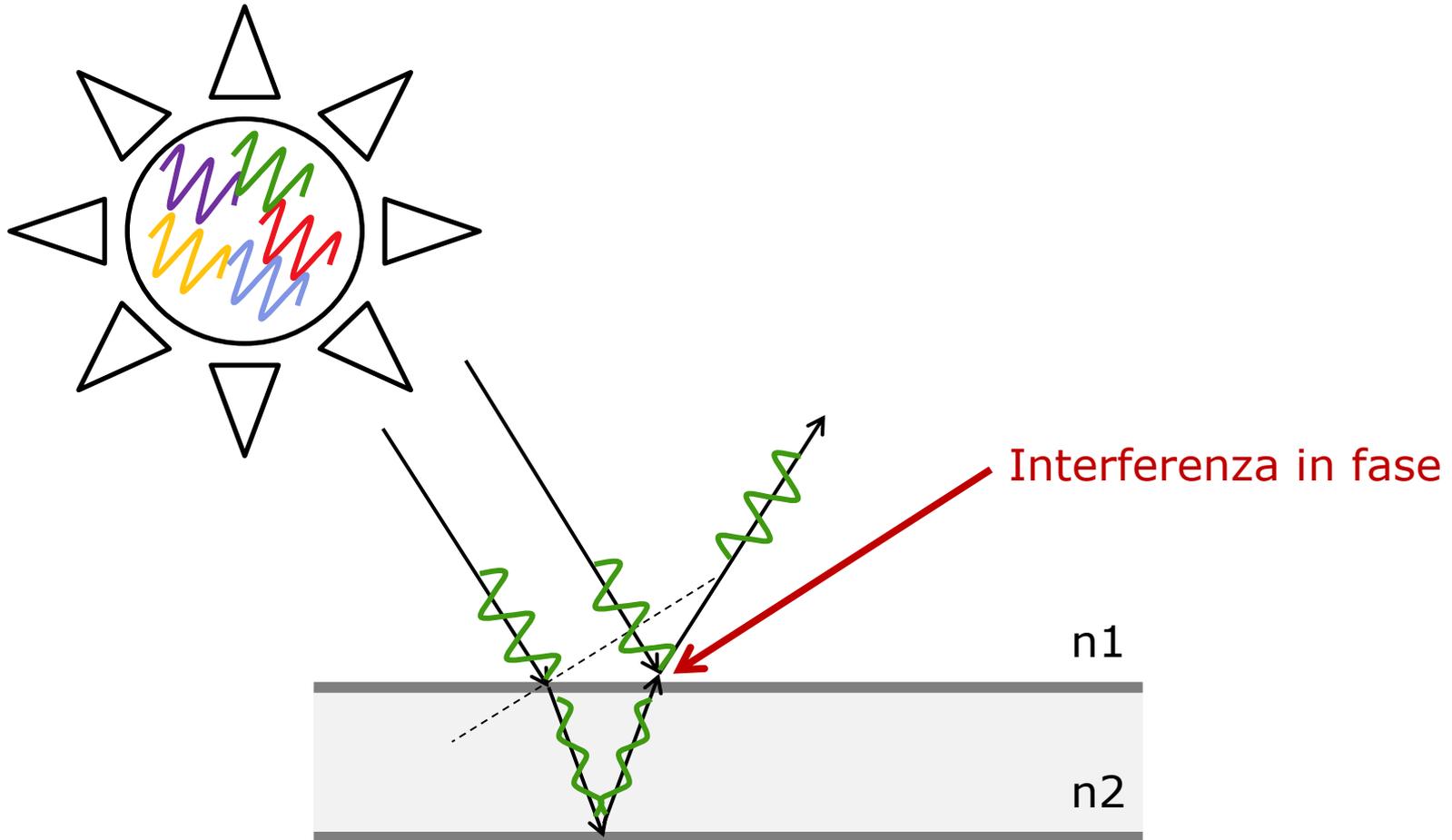


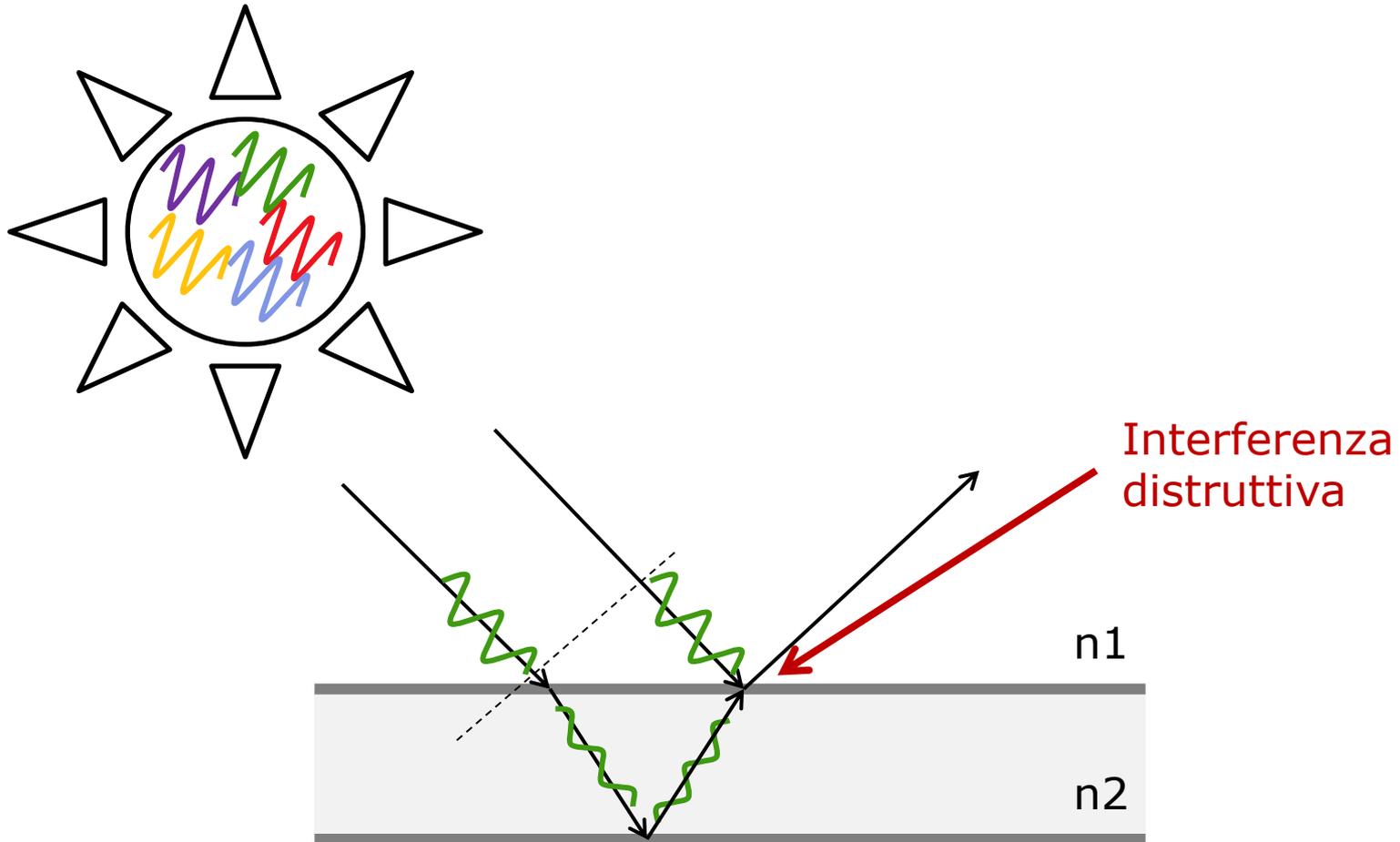
Cristalli fotonici di tutti i giorni...

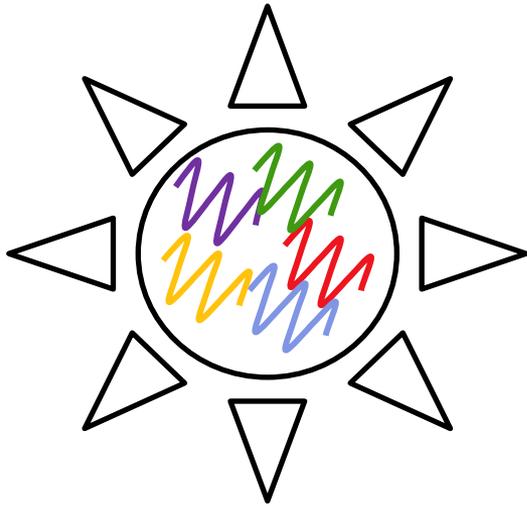




DIPENDE...

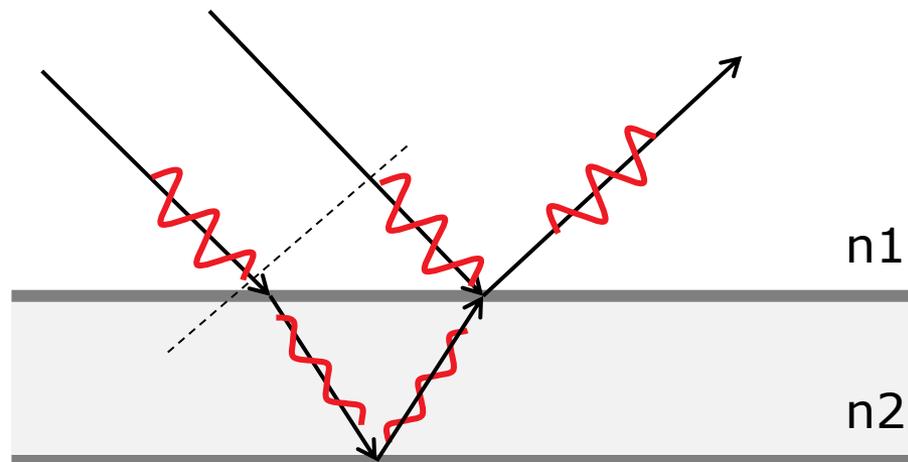


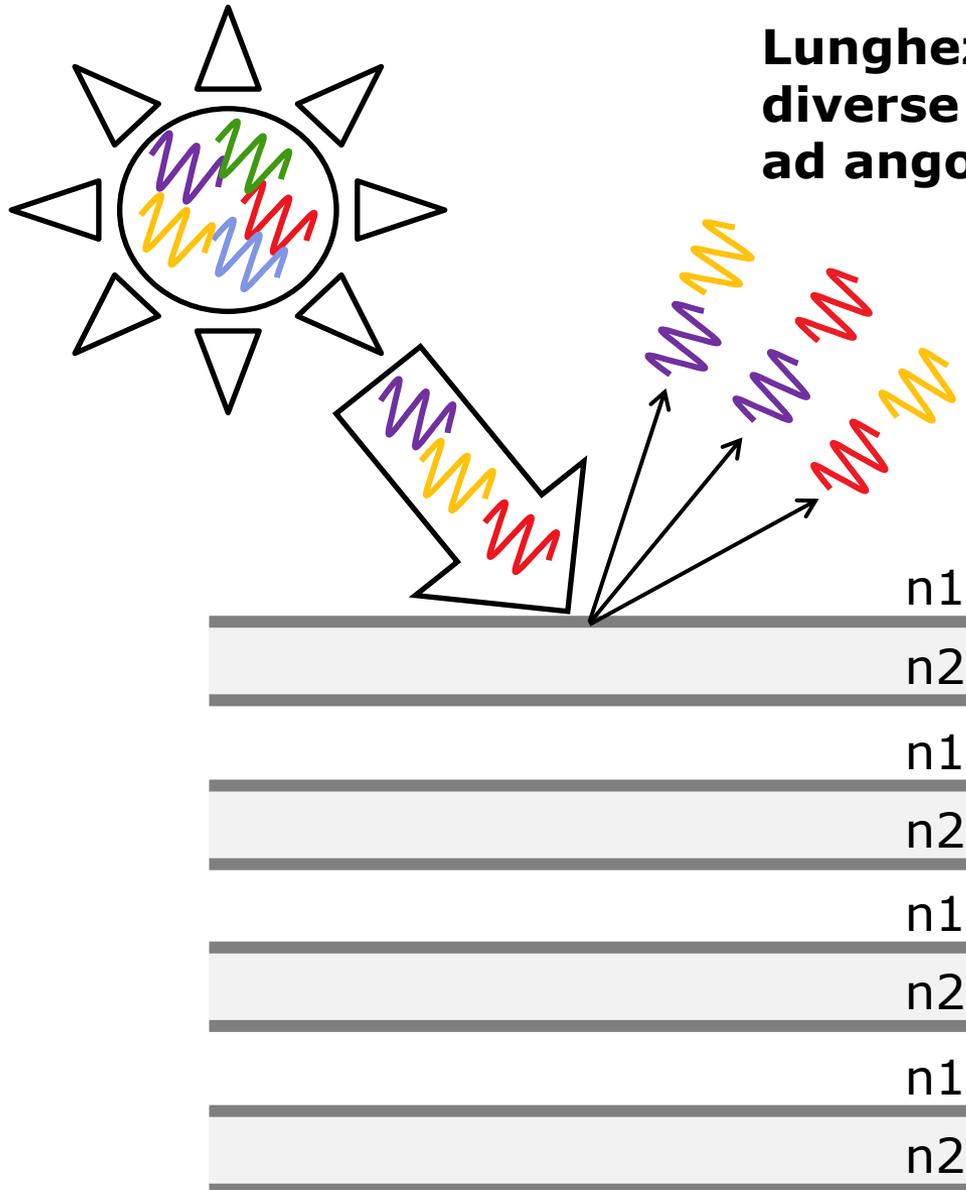




$$\lambda_{\text{rosso}} \neq \lambda_{\text{verde}}$$

Interferenza
distruttiva ad angoli
di incidenza diversi





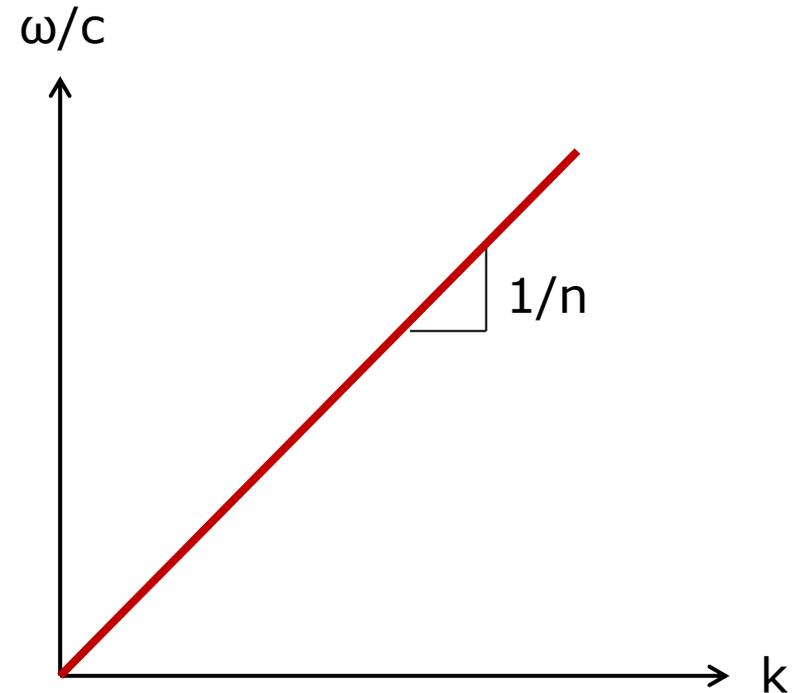
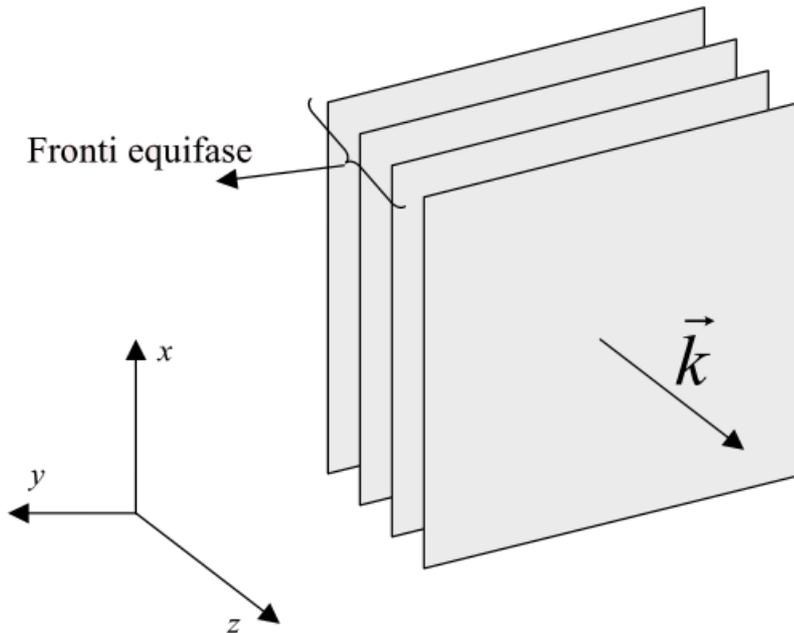
Lunghezze d'onda diverse *non riflesse* ad angoli diversi

Troppo selettivo per essere percepito

(variando di poco l'angolo di incidenza varia sensibilmente la lunghezza d'onda per cui si ha interferenza distruttiva)

Se uno strato non basta, mettiamone di più!

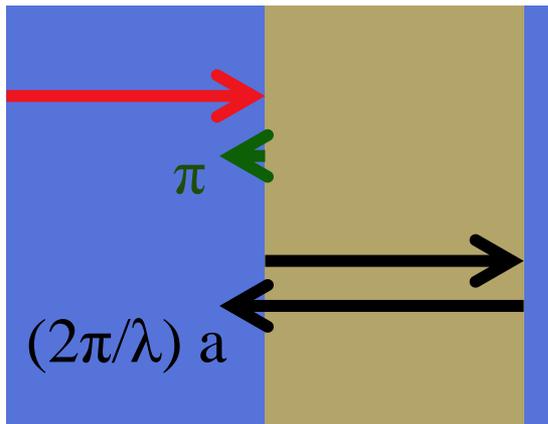
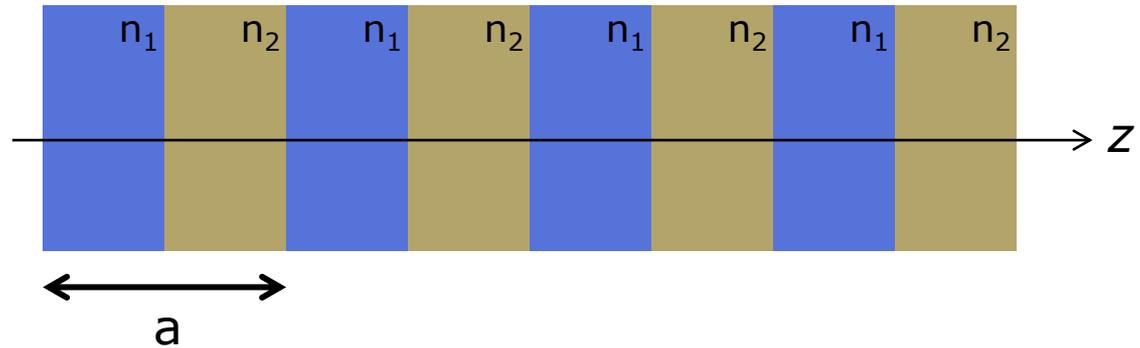
Onde piane in un mezzo omogeneo



$$\mathbf{E}(z) = \mathbf{E}(0)e^{-jkz}$$

$$k = n\omega/c = 2\pi/\lambda$$

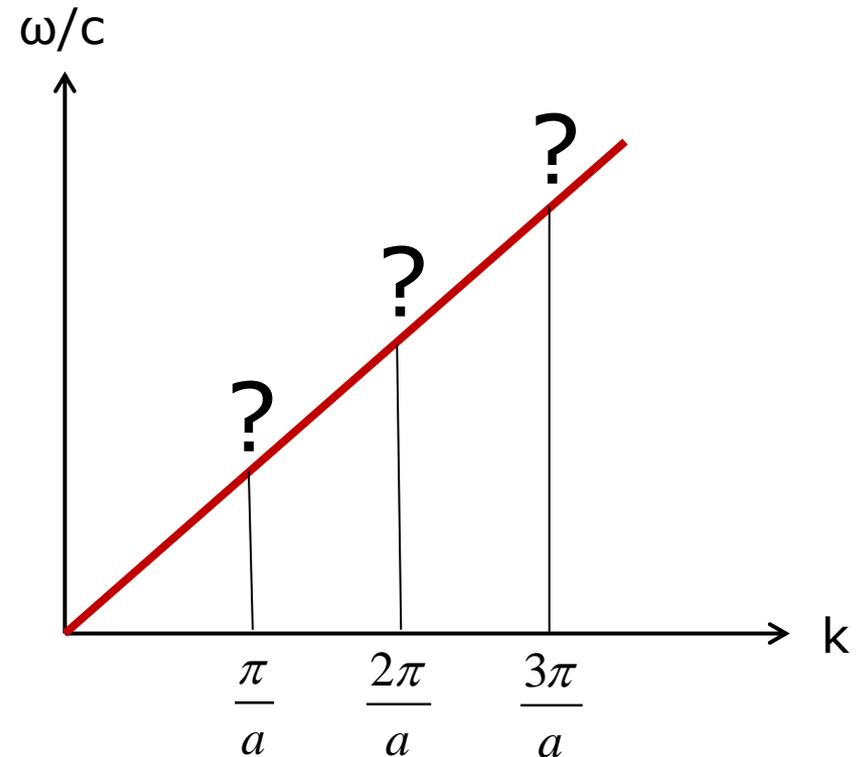
$$n(z) = n(z+n \cdot a)$$



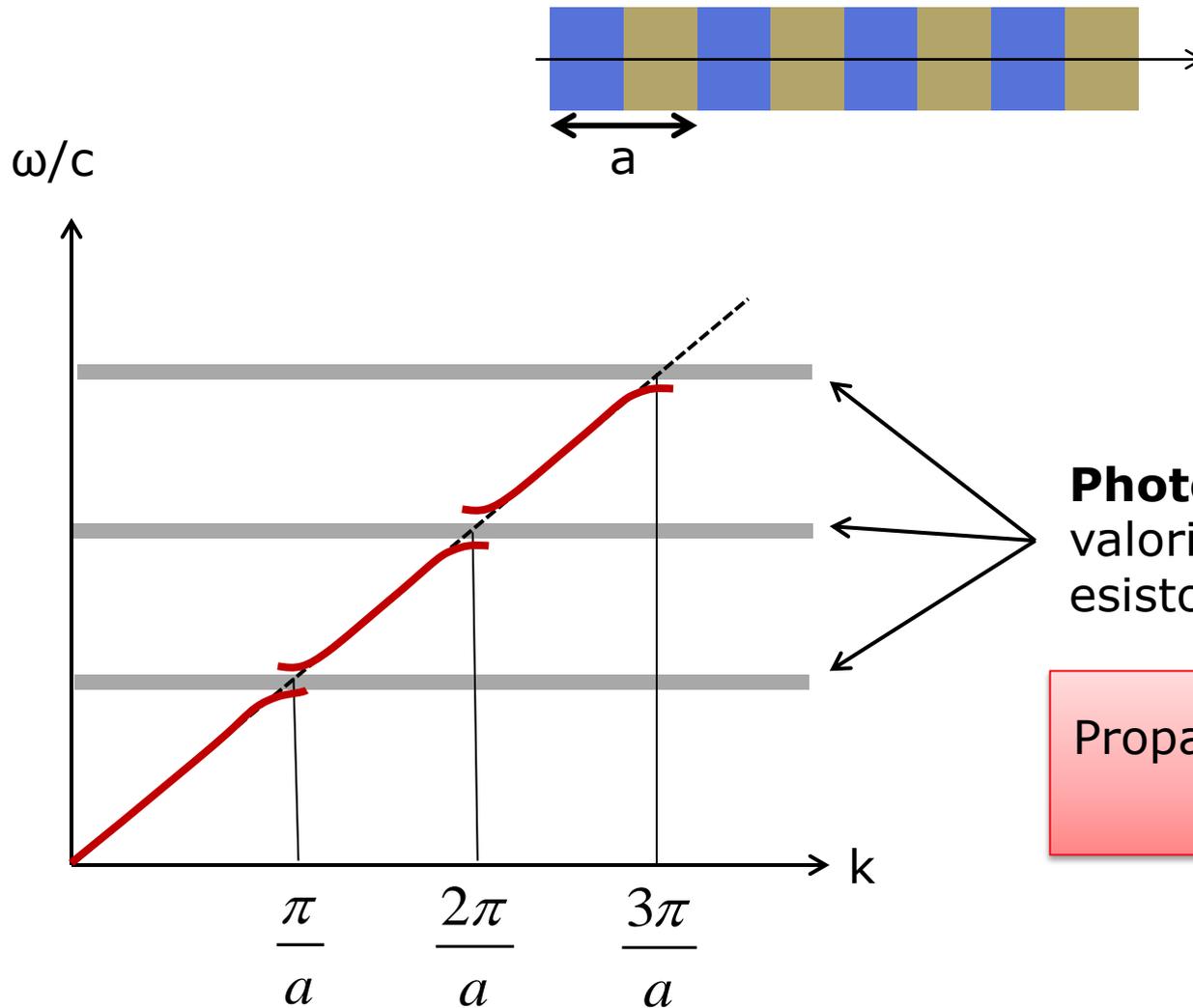
Condizione di Bragg:

$$a = m \lambda/2 \quad \leftrightarrow \quad k = m \pi/a$$

riflessione totale!



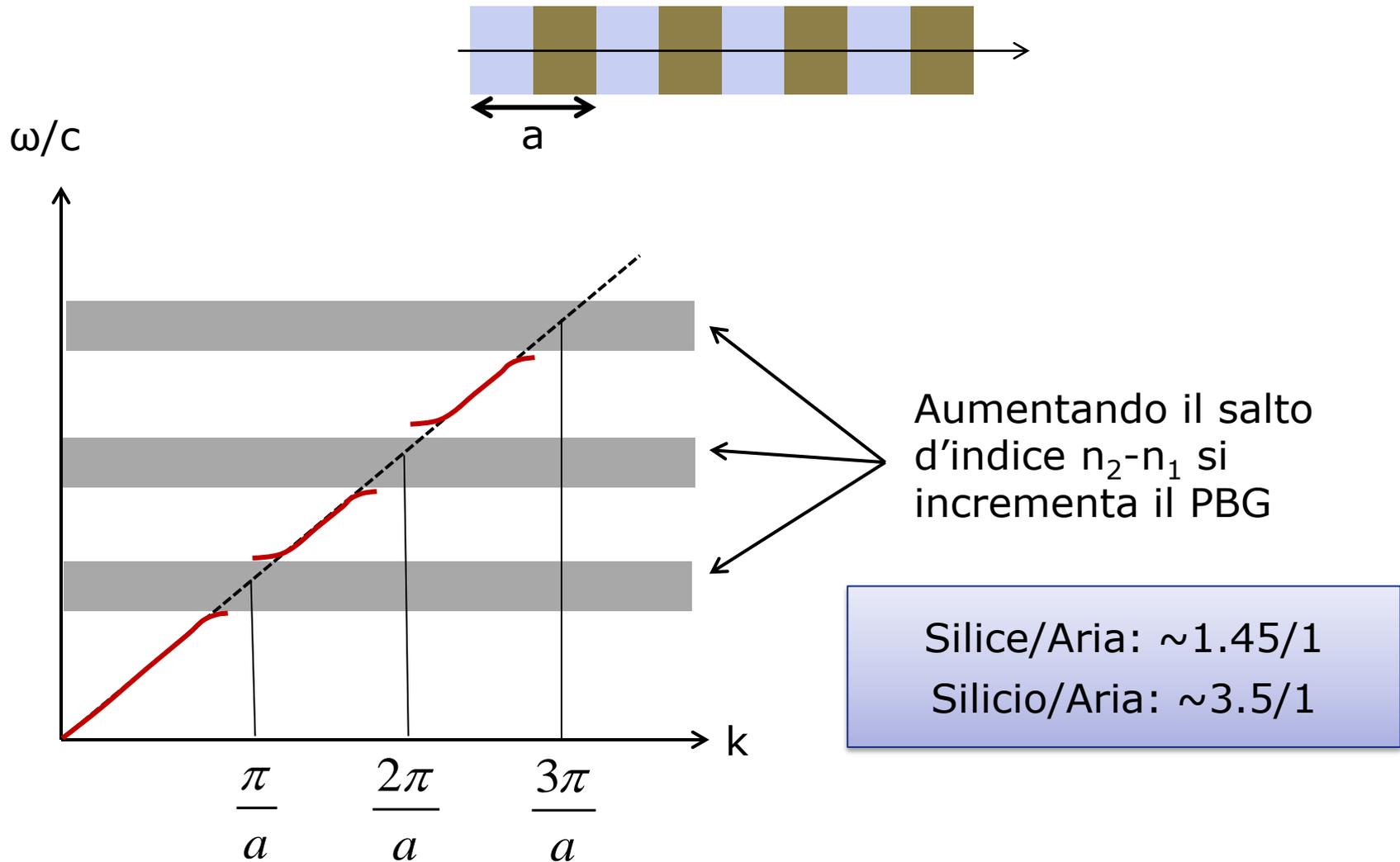
Photonic BandGap

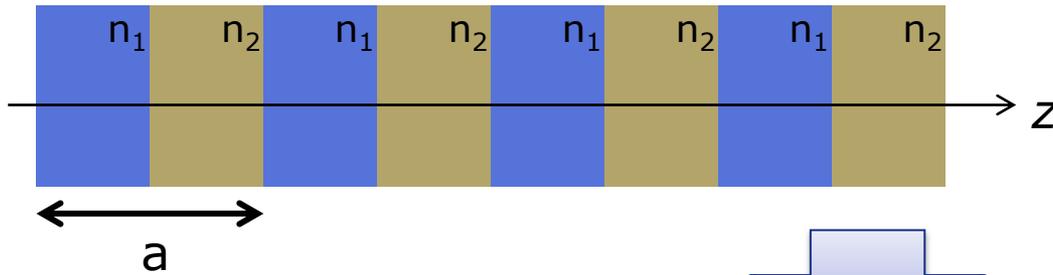


Photonic Band-gaps:
valori di ω per cui non
esistono k corrispondenti

Propagazione in direzione
+z impossibile!

Photonic BandGap





mezzo periodico:
 $n(z) = n(z+a)$

$$\bar{\nabla} \times \bar{\mathbf{E}} = -j\omega \mu \bar{\mathbf{H}}$$

$$\bar{\nabla} \times \bar{\mathbf{H}} = j\omega \varepsilon \bar{\mathbf{E}}$$

Le equazioni di Maxwell non cambiano da z a $z+a$

$$\mathbf{E}(z) = \mathbf{E}_{0k}(0)e^{-jkz}$$

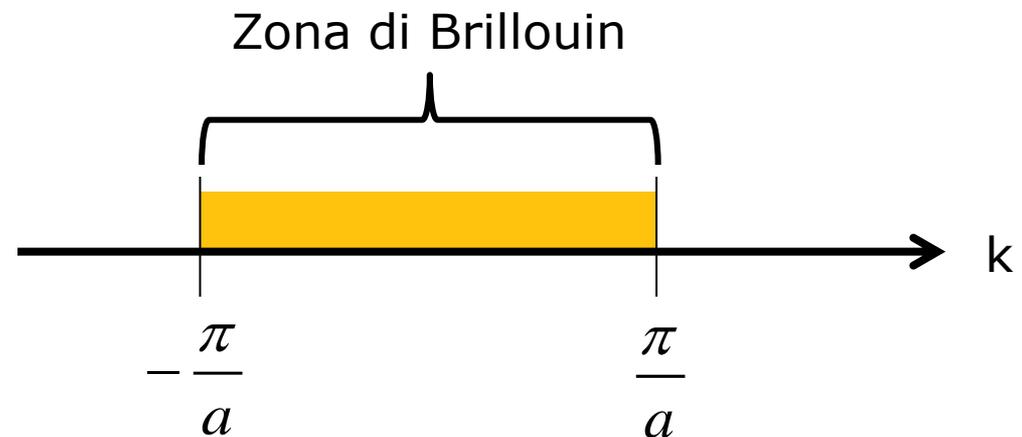
$$\mathbf{E}_{0k}(z) = \mathbf{E}_{0k}(z+a)$$

cioè: le soluzioni delle eq. di Maxwell devono essere periodiche in z con periodo a

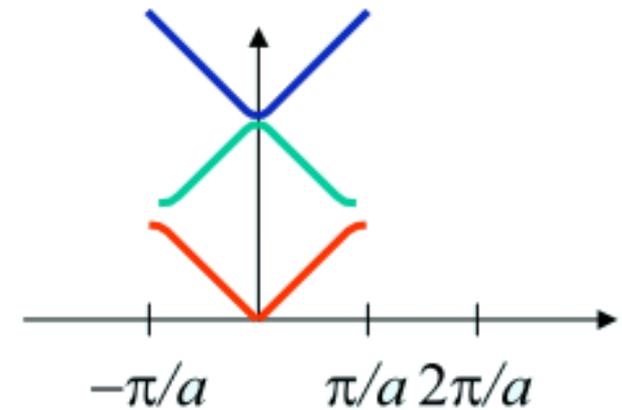
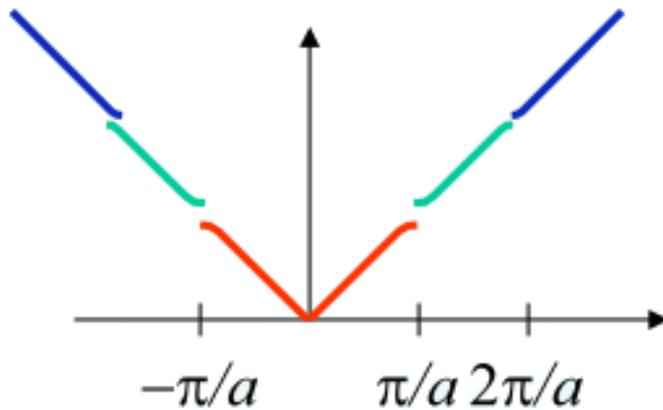
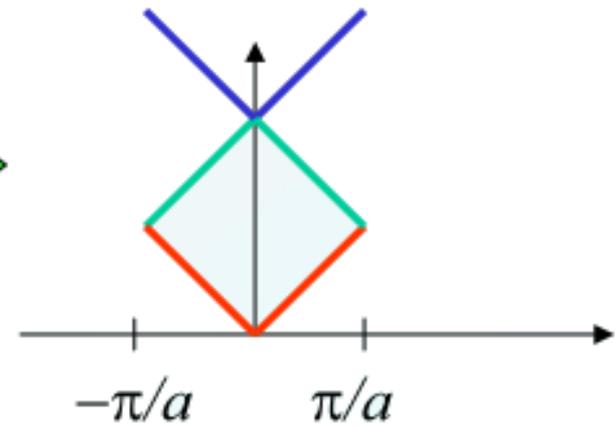
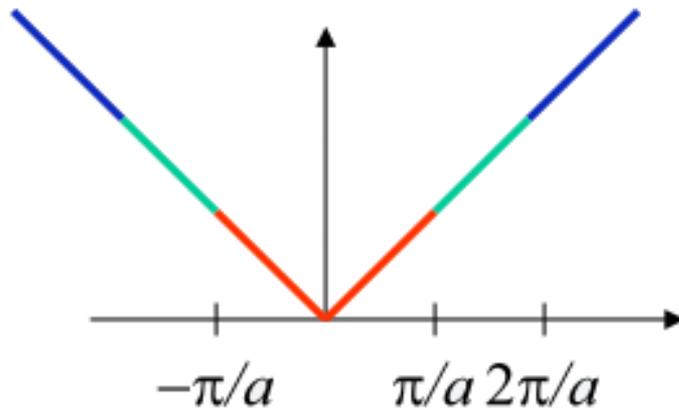
- Le soluzioni con k che differiscono per multipli di $2\pi/a$ sono identiche
- Le soluzioni corrispondenti devono avere la stessa frequenza:

$$\omega(k) = \omega(k + m\pi/a)$$

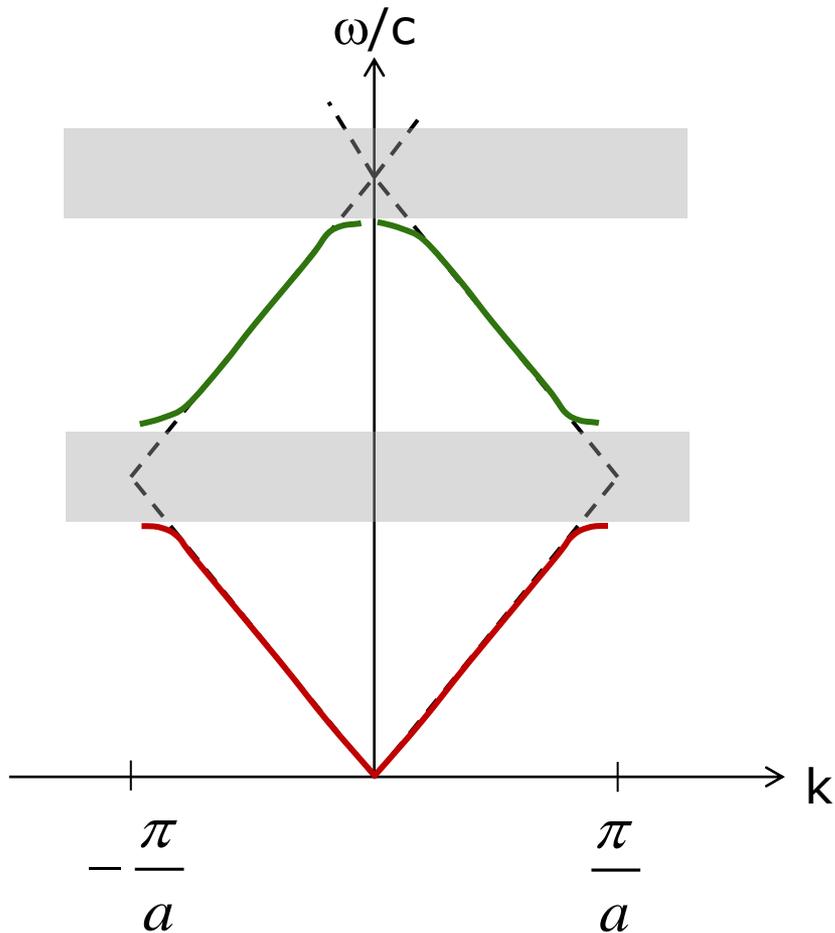
La regione di k corrispondenti a soluzioni non ridondanti si chiama zona di Brillouin



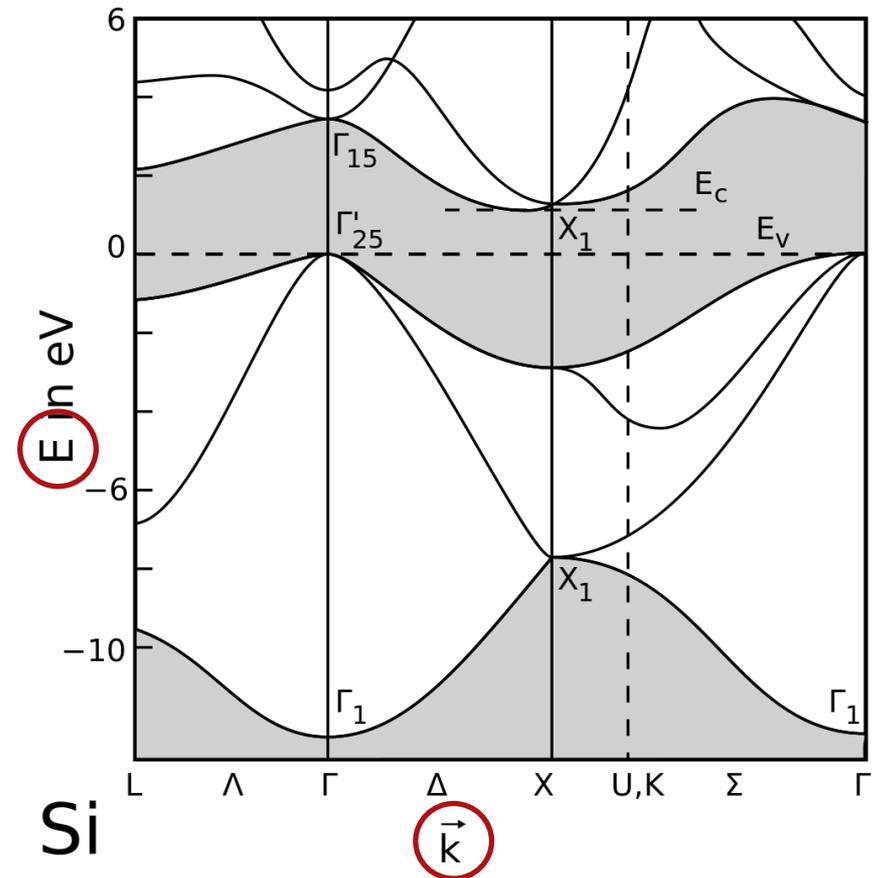
Le curve di dispersione vengono tracciate solo nella zona di Brillouin

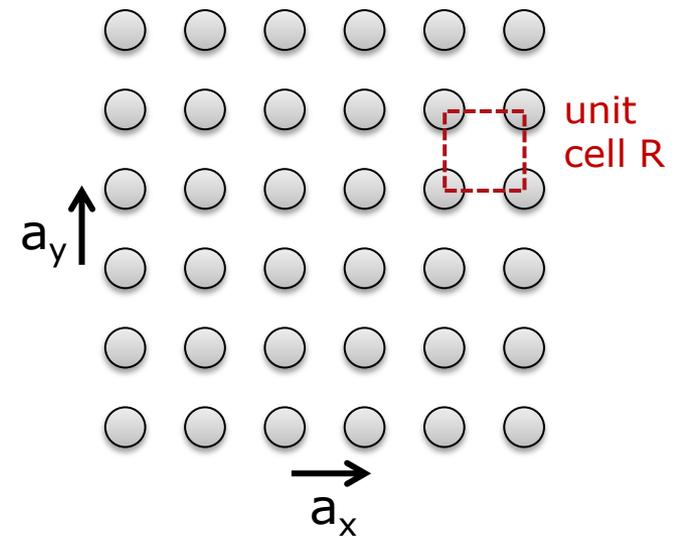
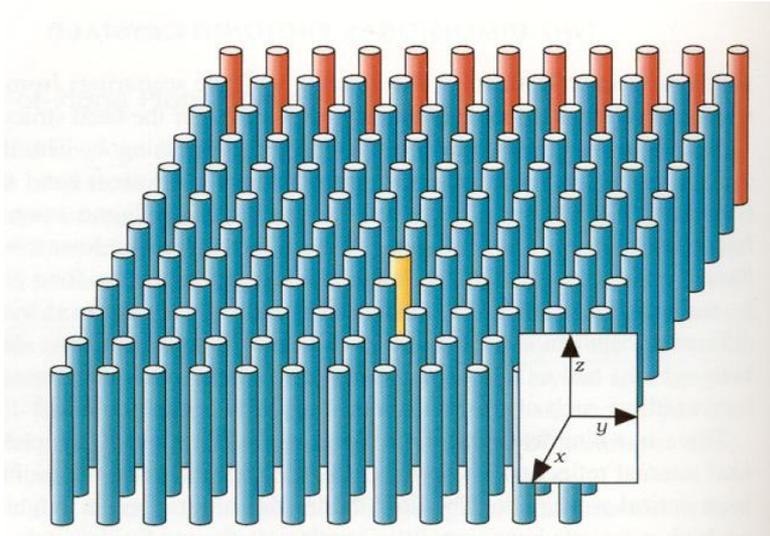


Photonic Bandgap



Semiconductor bandgap

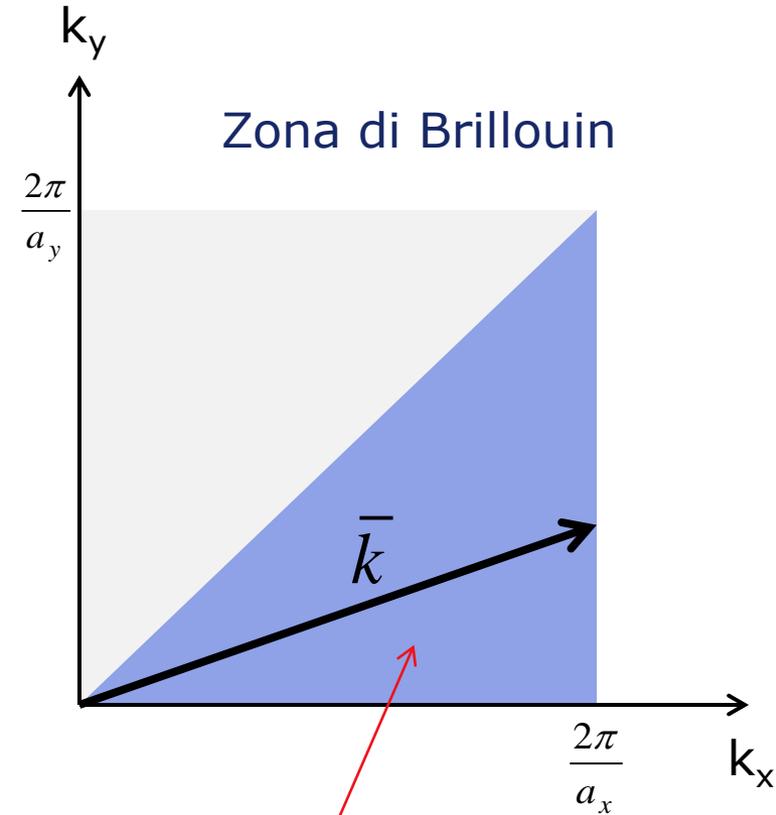
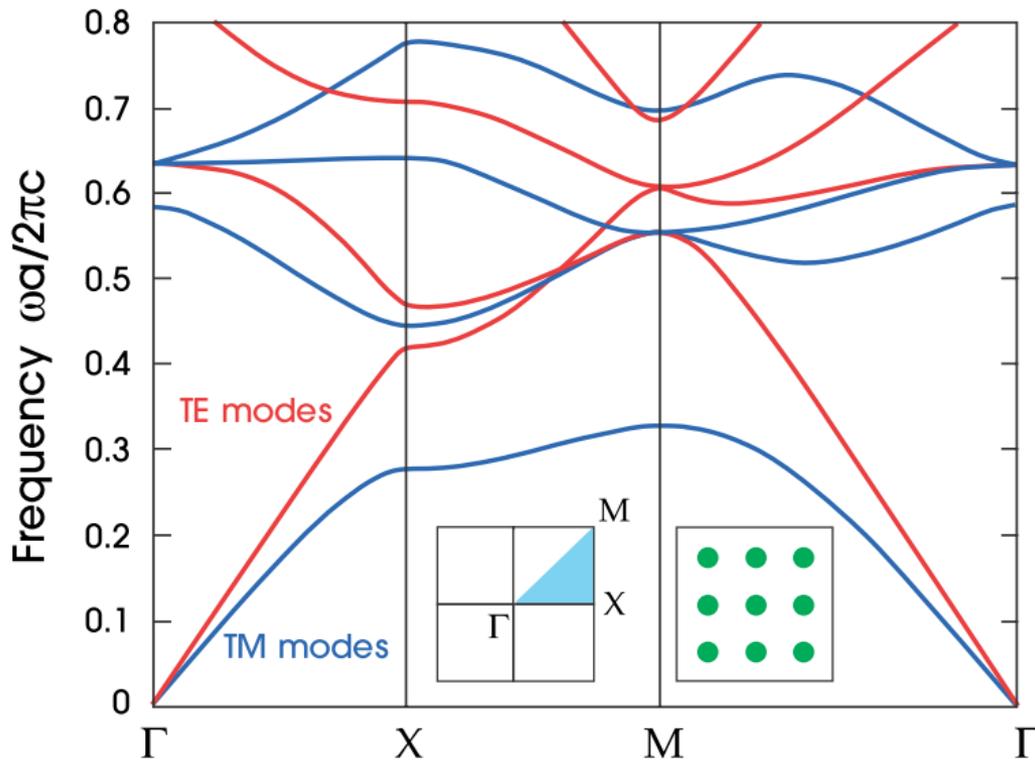




- Un cristallo bidimensionale è caratterizzato da una cella elementare R e da un vettore di reticolo:

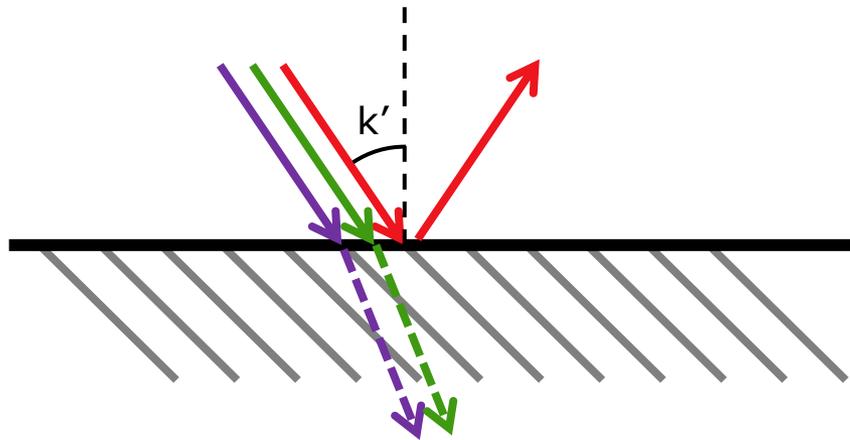
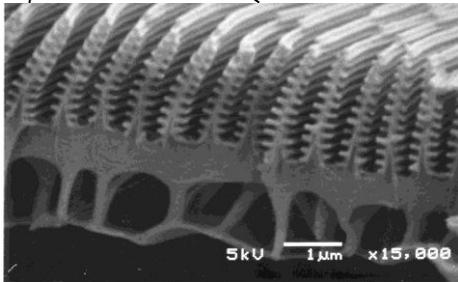
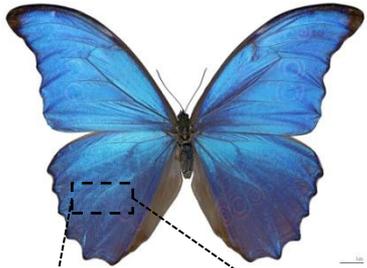
$$\bar{a} = a_x \hat{i} + a_y \hat{j}$$

- Ancora una volta è possibile definire una zona di Brillouin, dipendente da a_x e a_y , che contiene i vettori \mathbf{k} a cui corrispondono soluzioni non ridondanti delle equazioni di Maxwell



Zona di Brillouin
irriducibile

Tornando alla *Morpho*

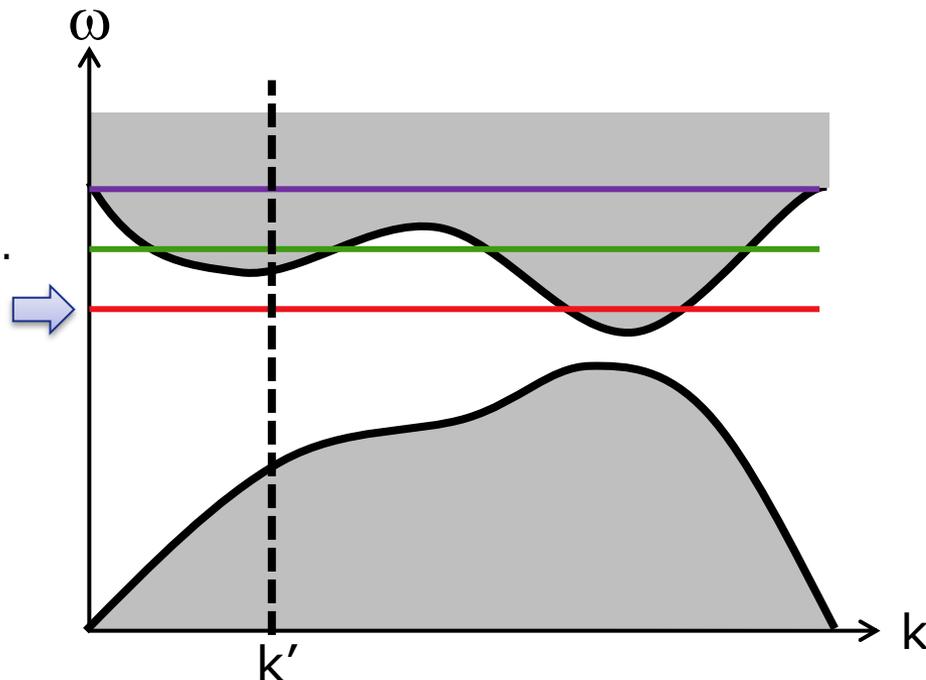


← superficie dell'ala

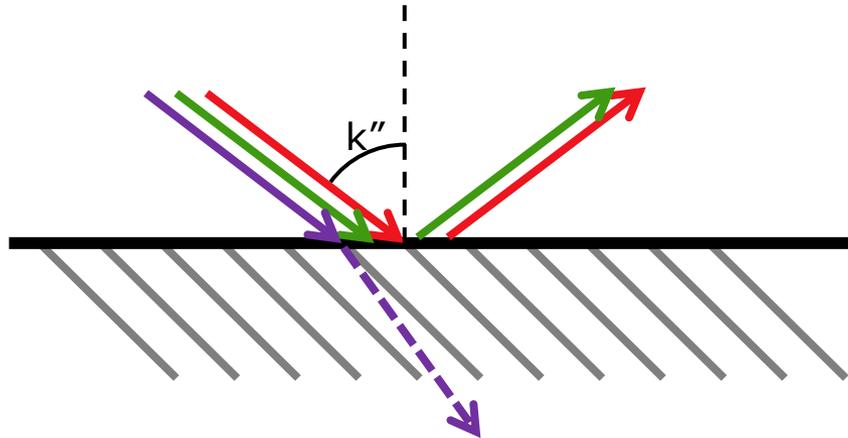
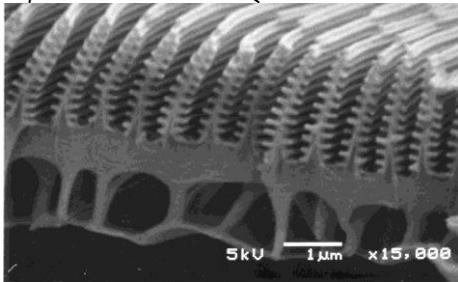
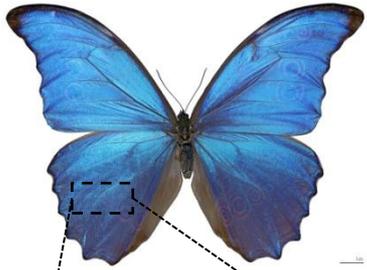
← "cristallo fotonico"

A $k=k'$ il **rosso** è nel PBG.

Non può propagarsi attraverso l'ala e viene riflesso.



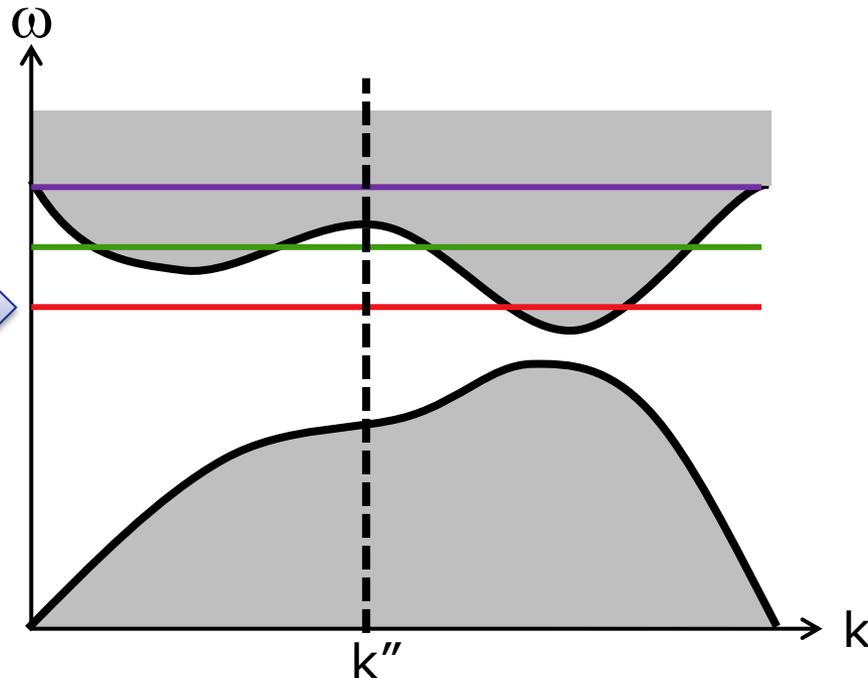
Tornando alla *Morpho*

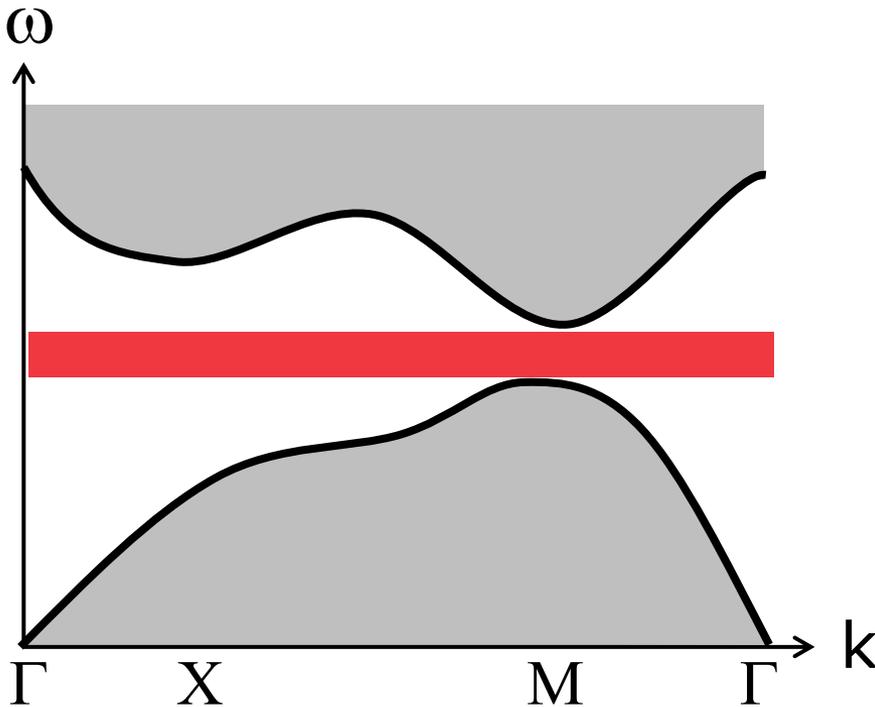


← superficie dell'ala

← "cristallo fotonico"

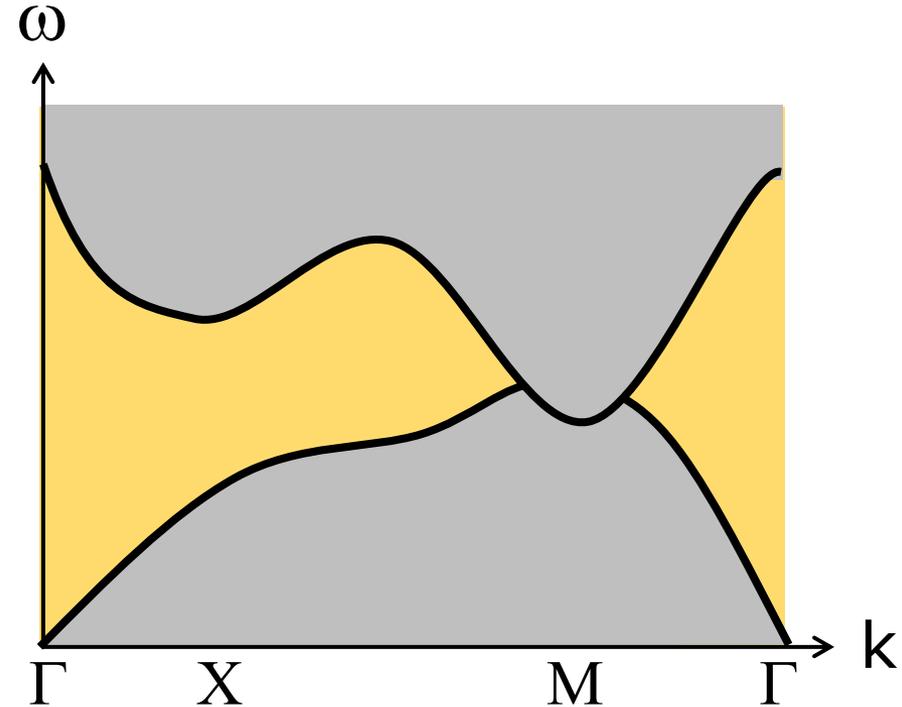
A $k=k''$ **rosso e verde sono nel PBG** e vengono riflessi.





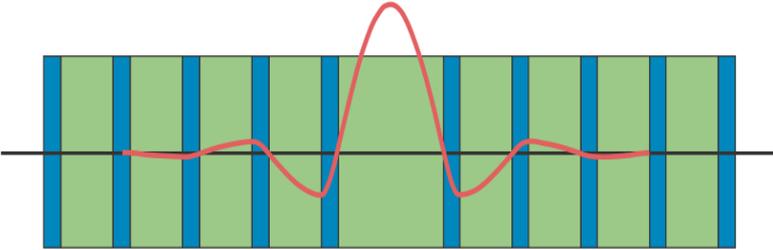
Bandgap completo

Esiste un range di frequenze in cui la radiazione è riflessa indipendentemente dall'angolo di incidenza

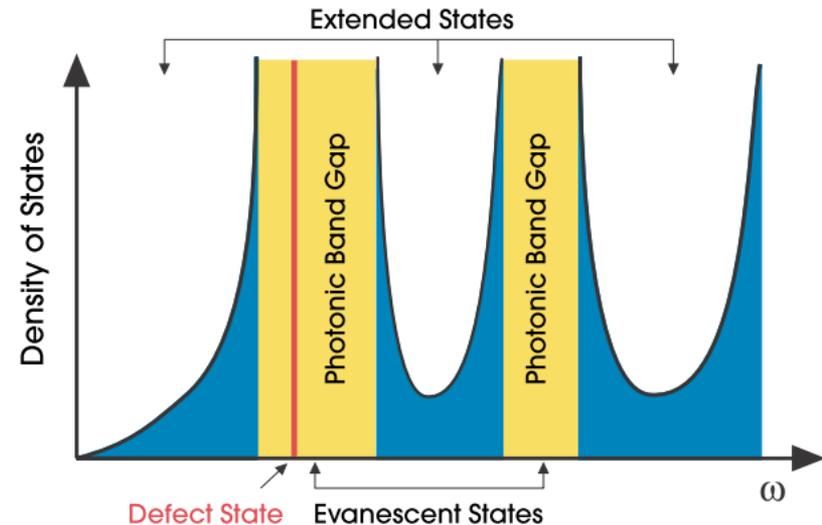
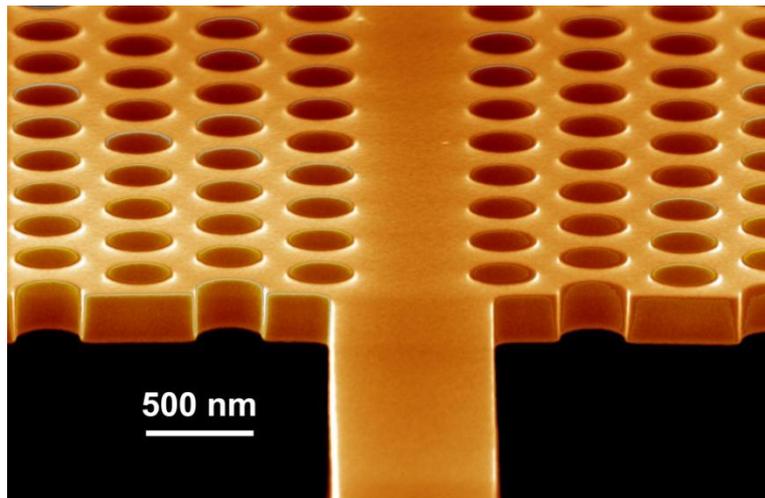


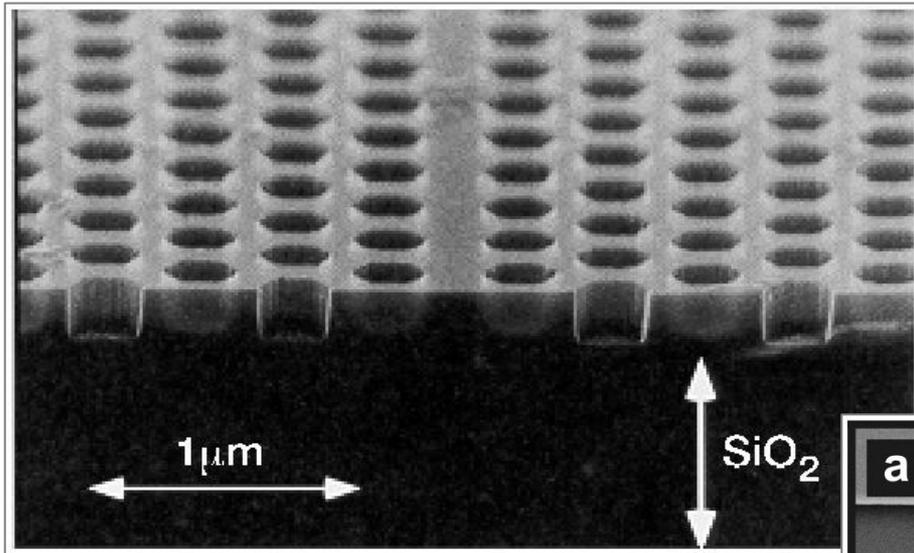
Bandgap incompleto

La radiazione è completamente riflessa solo per alcune combinazioni di (ω, k)



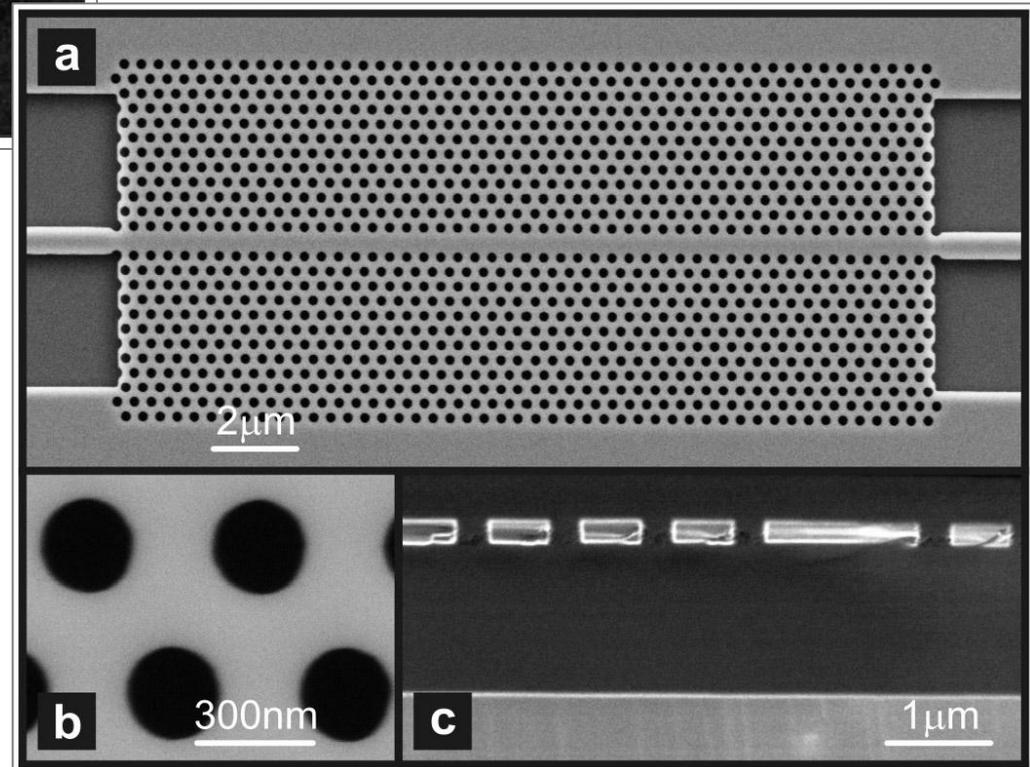
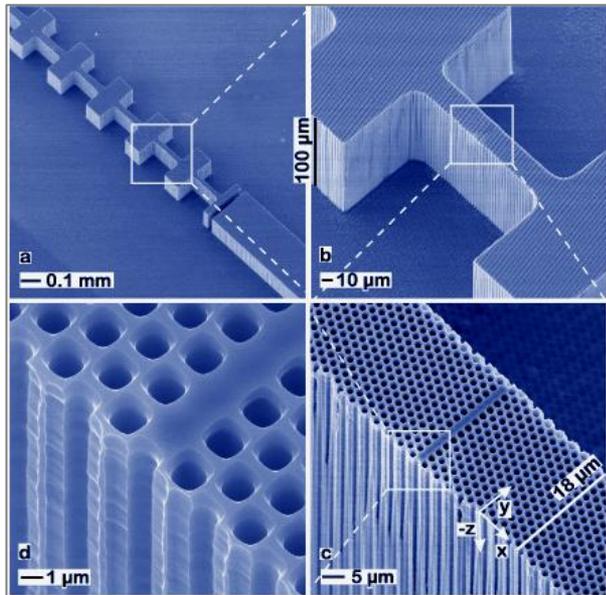
- Interrompendo la periodicità si crea un difetto
- L'effetto del difetto è consentire l'esistenza di una soluzione all'interno del bandgap
- Il campo e.m. relativo a quella soluzione non può propagarsi nel reticolo, ed è quindi **confinato** nel difetto

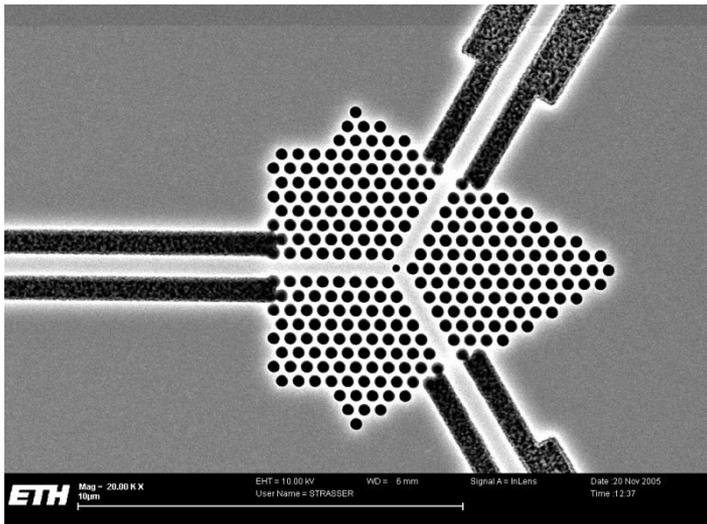
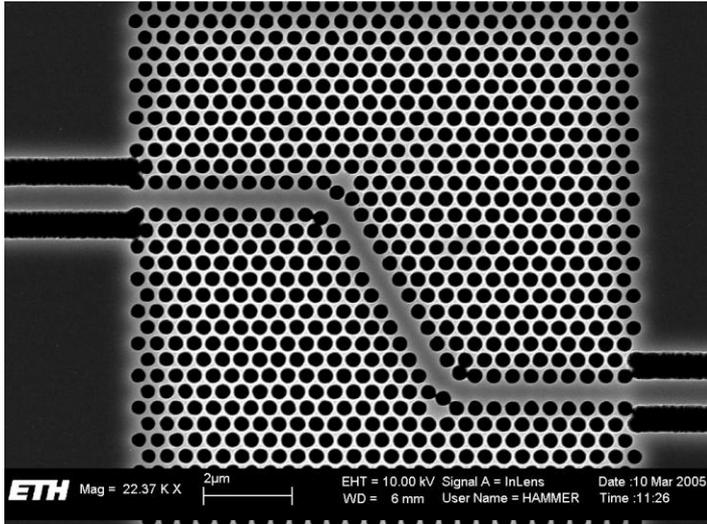




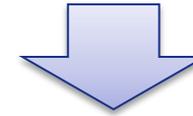
Ottica integrata

Sensori / biosensori

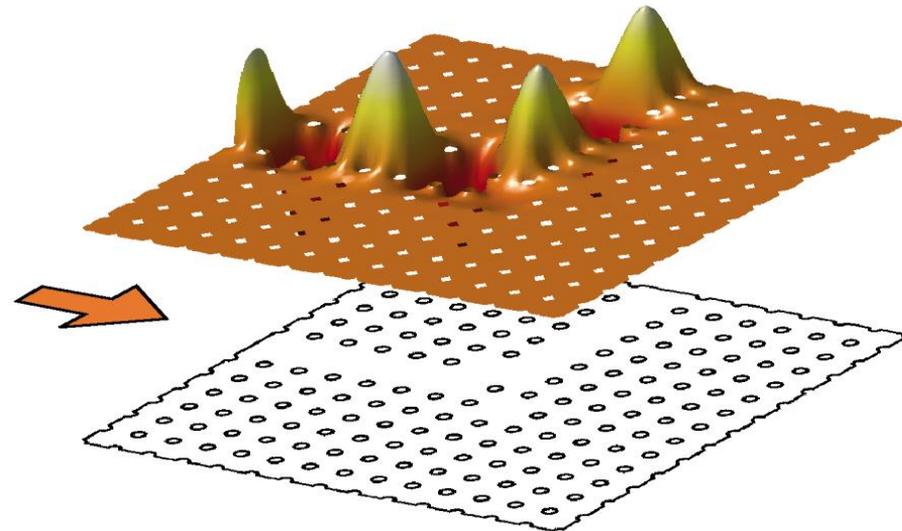


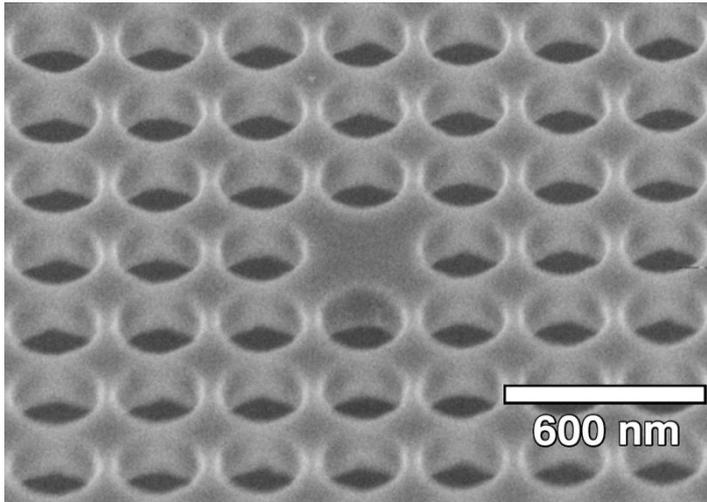
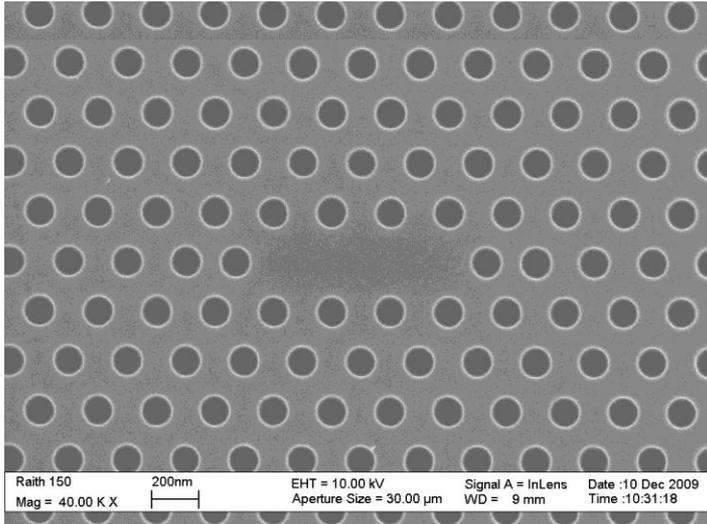


- Se il cristallo fotonico ha un PBG completo a una certa λ , il campo e.m. è confinato nel difetto per qualunque angolo di incidenza

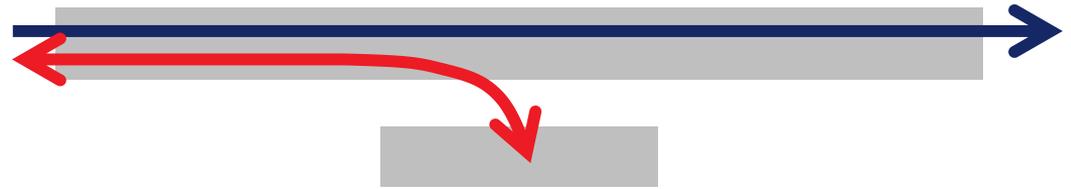


Assenza di perdite dovute ai modi irradianti

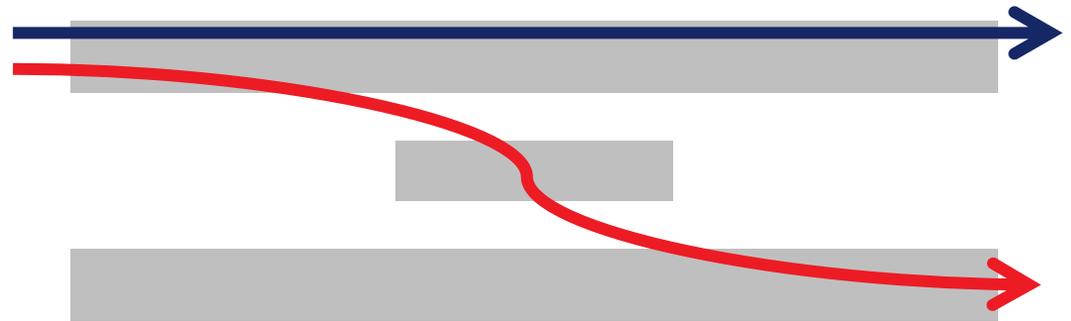




- Difetti puntuali possono essere utilizzati per realizzare cavità risonanti o filtri



Accoppiamento guida-cavità

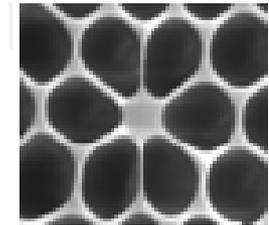
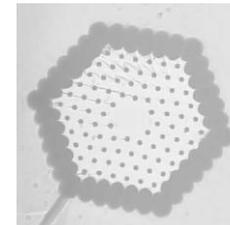
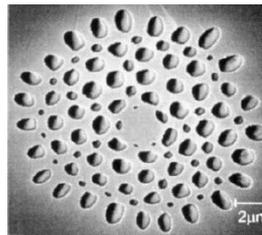
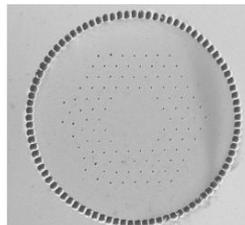
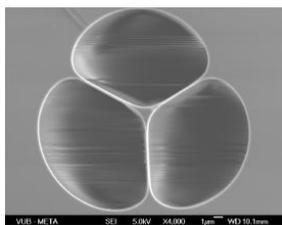
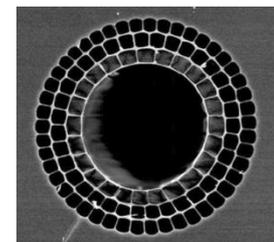
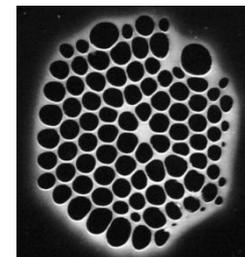
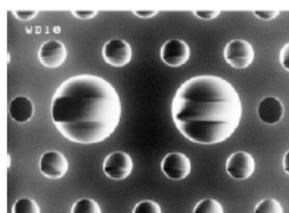
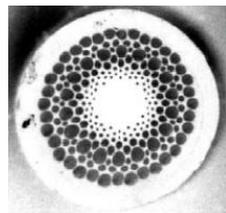
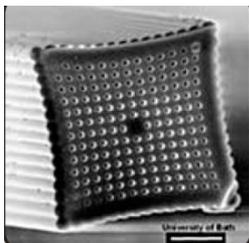
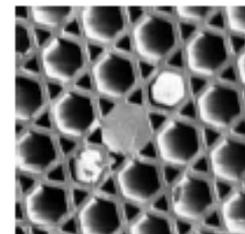
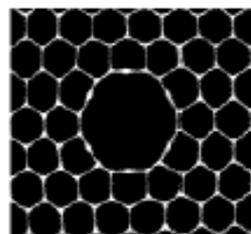
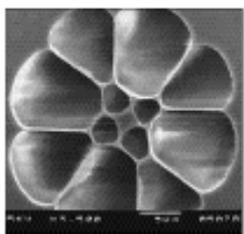
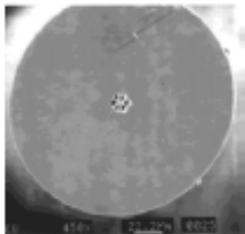
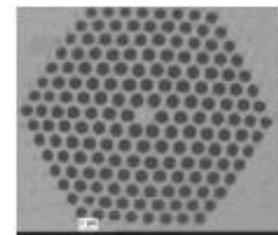
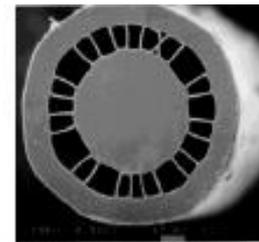
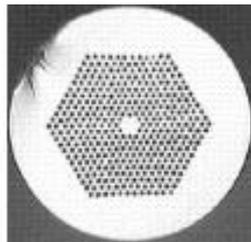
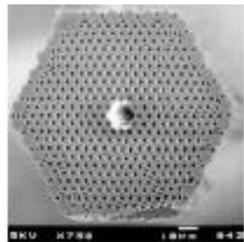


Accoppiamento guida-cavità-guida

FIBRE A CRISTALLO FOTONICO

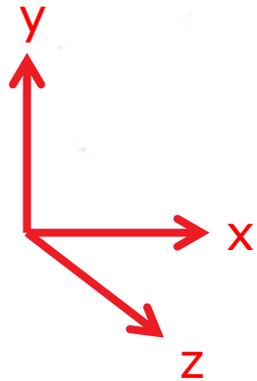


Fibre a cristallo fotonico (PCF)



Cristallo fotonico sul piano xy

Propagazione in direzione z



20KV

10µm
X1,800

36mm

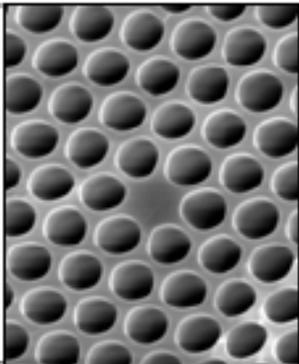
L01

Overall terms:

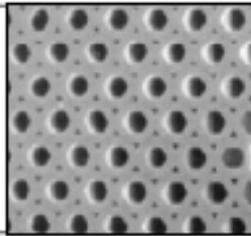
- Photonic Crystal Fibre (PCF)
- Microstructured Fibre (MSF)
- Microstructured Optical Fibre (MOF)

Main classes:

- High-Index Core
- Index-Guiding
- Holey
- Hole-assisted

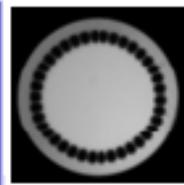


- Photonic Band Gap (PBG)
- Bandgap Guiding

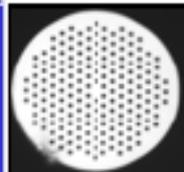


Subclasses:

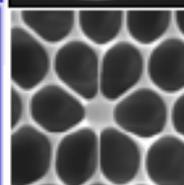
High NA



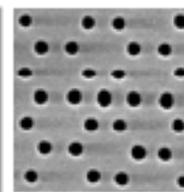
Large Mode Area



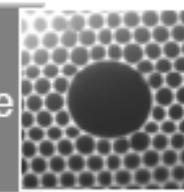
Highly Nonlinear



Low-index Core



Air Guiding Hollow Core

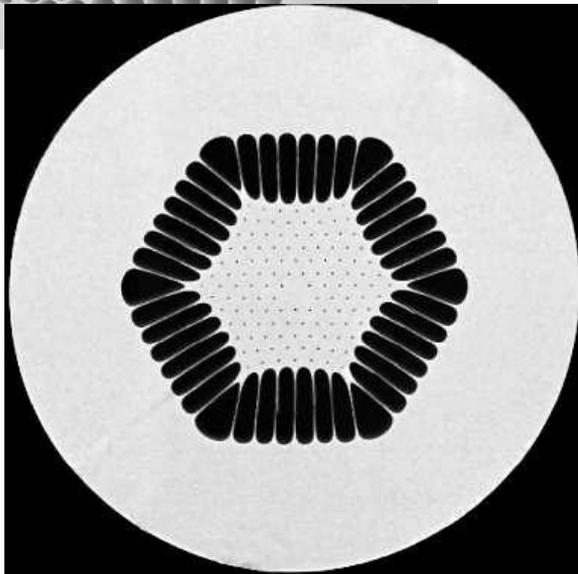
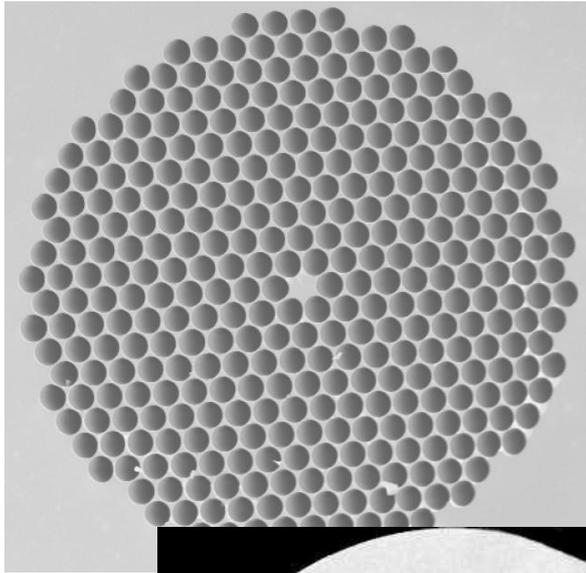


Outline:

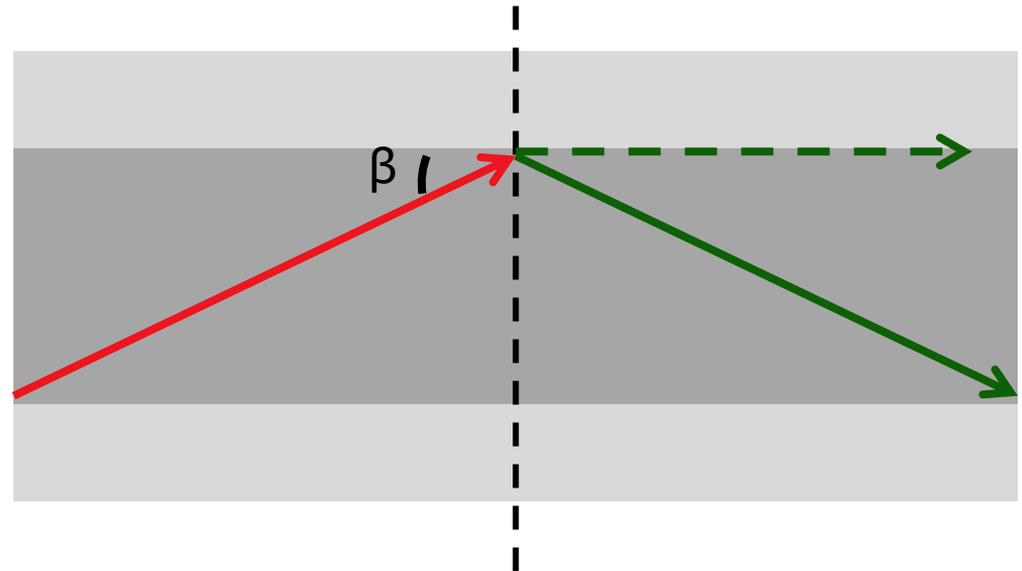
- [Fundamentals](#)
- Fabrication
- Methods
- Fibre types
- Applications
- Progress
- Perspectives

Reference:

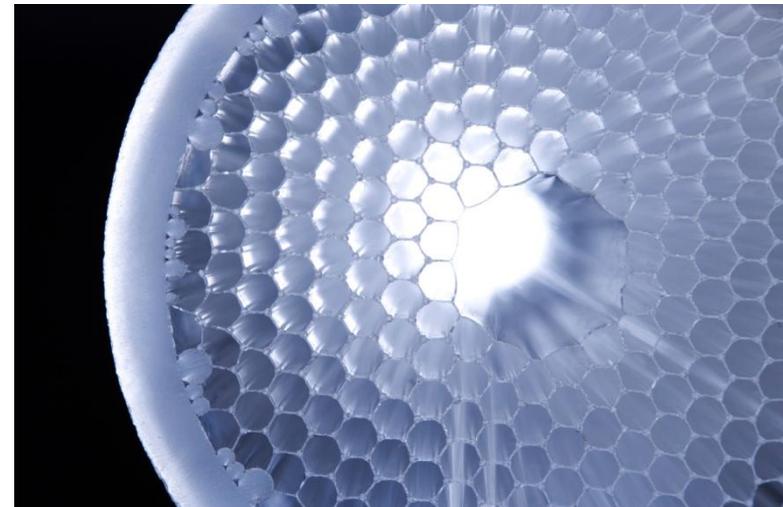
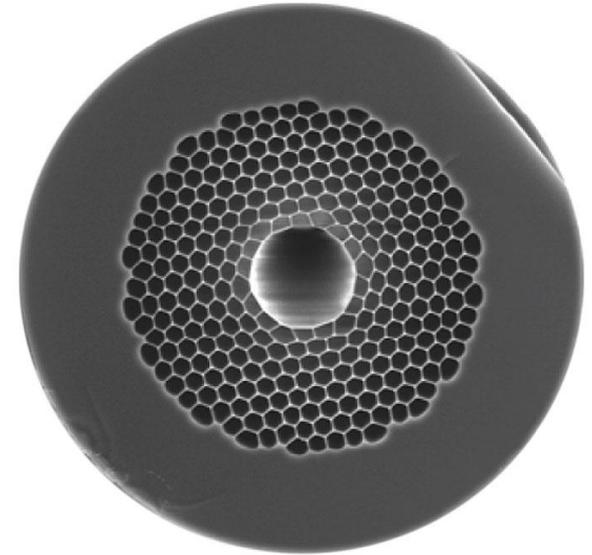
Photos supplied by Crystal Fibre A/S



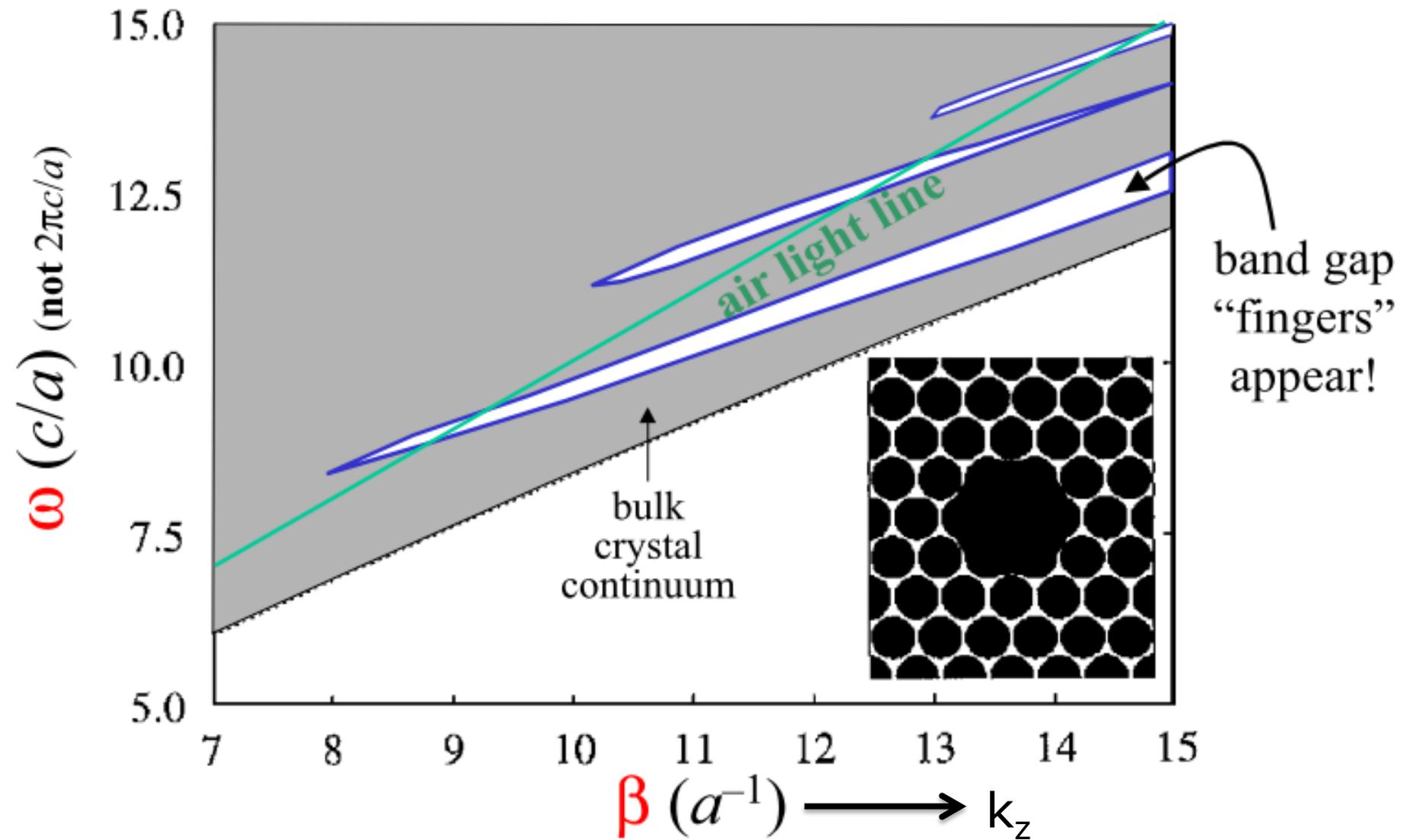
- La presenza dei fori nel cladding dà luogo a un'indice di rifrazione *mediamente* più basso di quello del core.
- Il campo è confinato per effetto della **riflessione interna totale** (modificata)

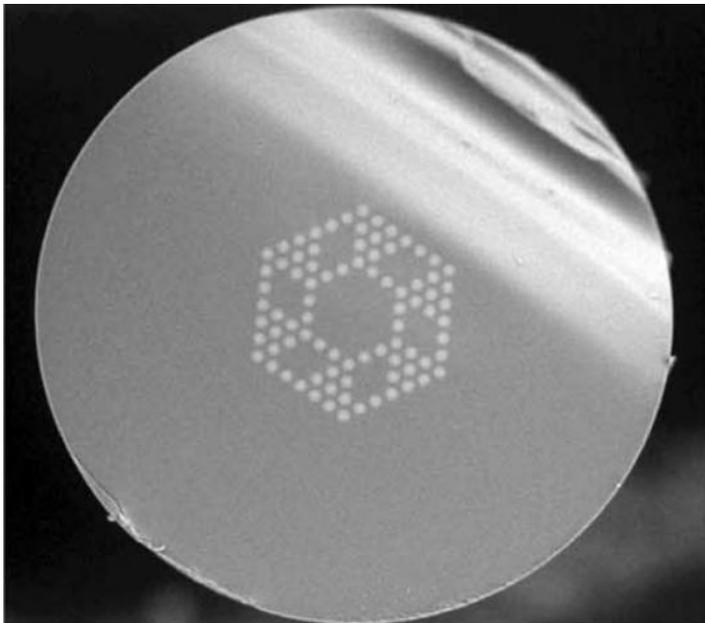
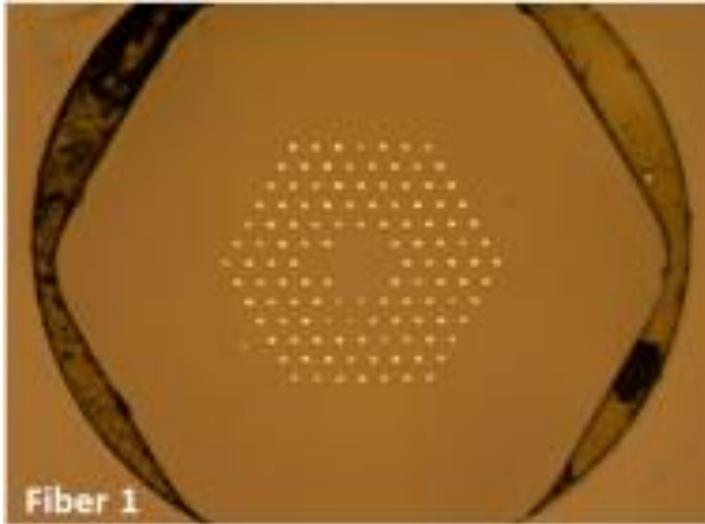


- Confinano il campo in un nucleo cavo, in **aria** ($n = 1$)
- Modi guidati solo per valori di lunghezza d'onda $\lambda = 2\pi c/\omega$ per cui esiste un PBG completo (i.e.: per ogni k_t) sul piano trasverso



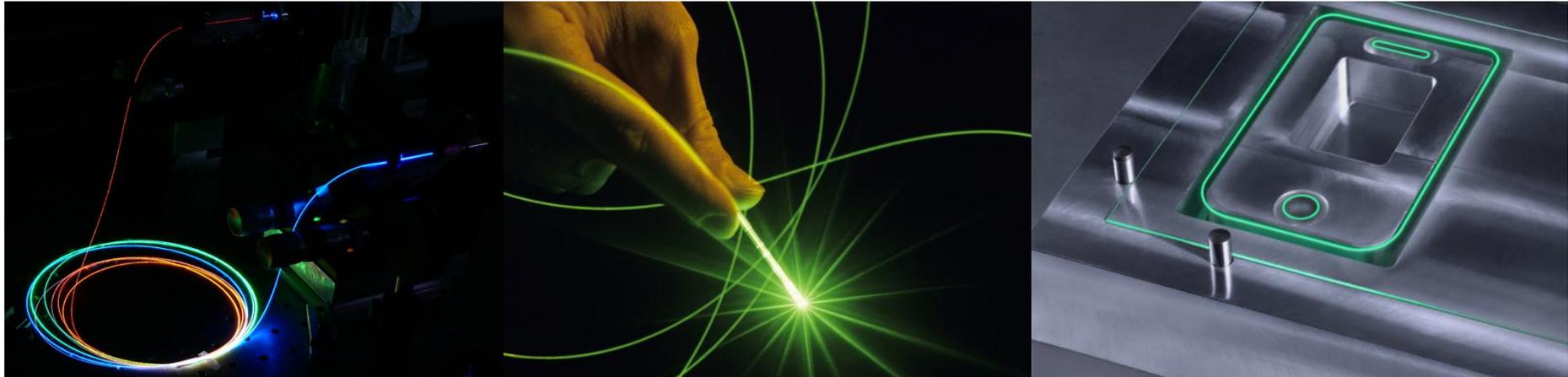
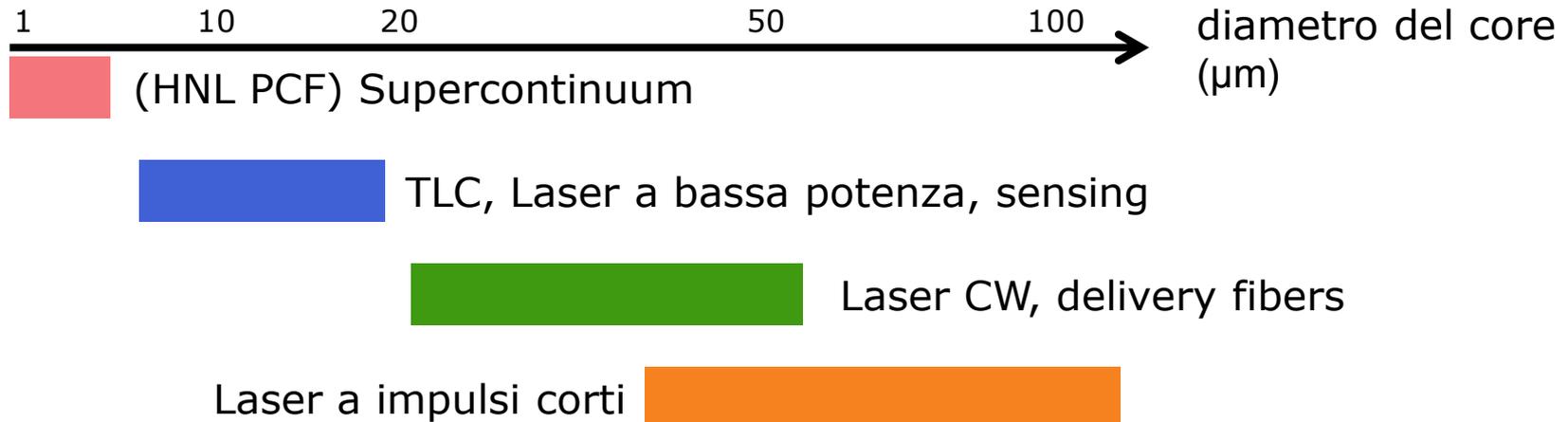
Photonic bandgap in PCFs





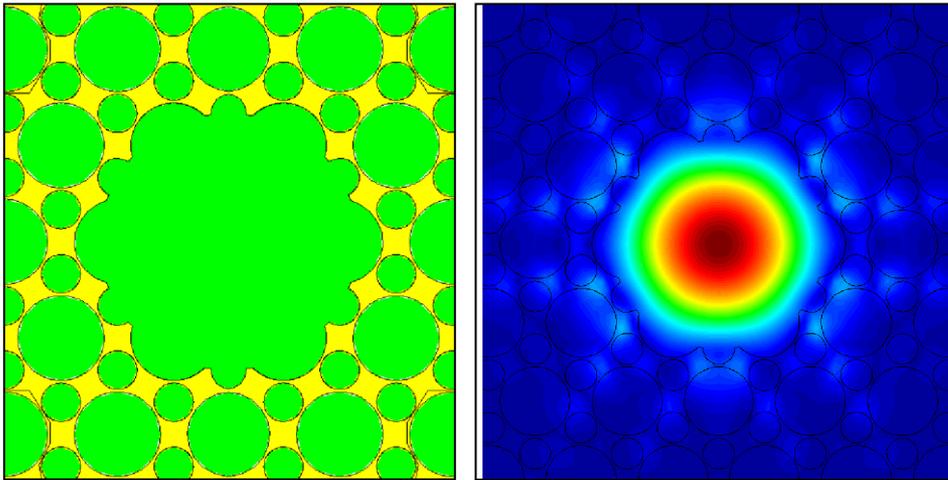
- Possibile ottenere guidaggio sul piano trasverso mediante PBG anche con salto d'indice basso ($\sim 2\%$)
- Eliminare i fori può facilitare l'utilizzo della PCF (i.e. giunti, impurità, danneggiamento, facilità di realizzazione...)

PCF a core solido



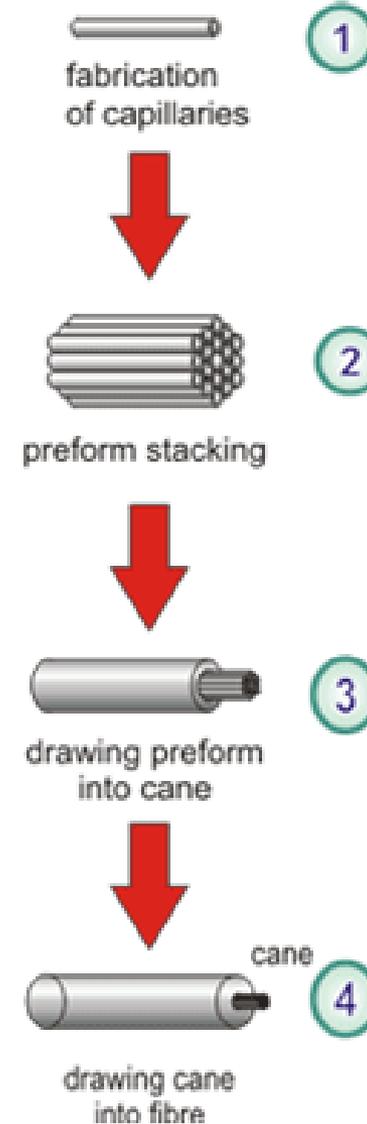
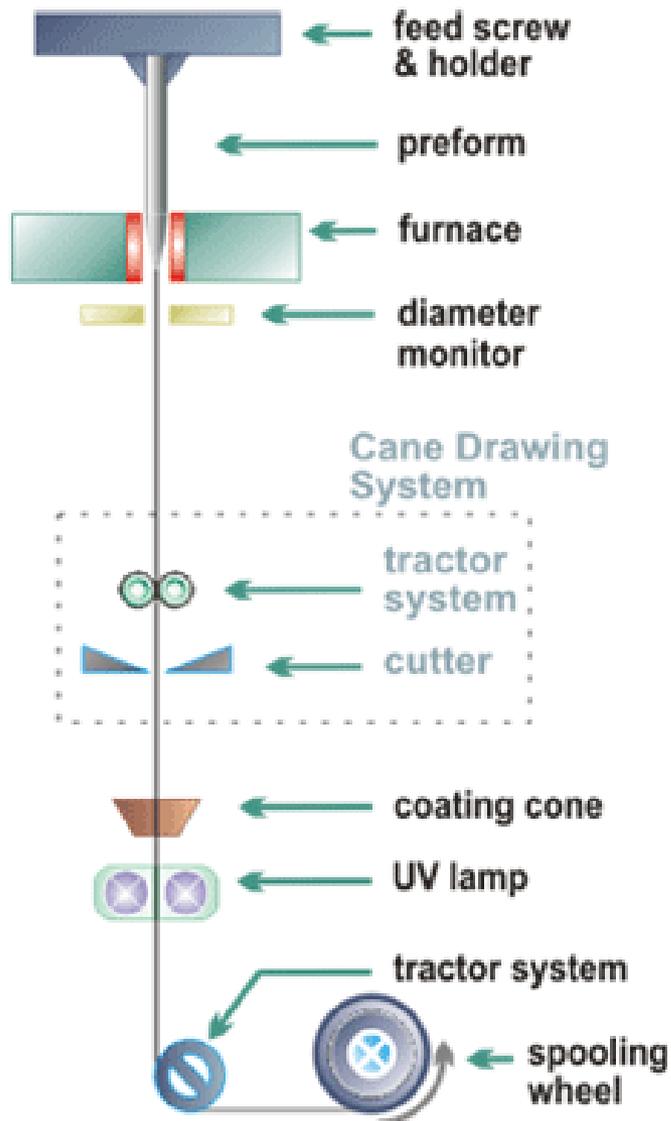
PCF a core cavo

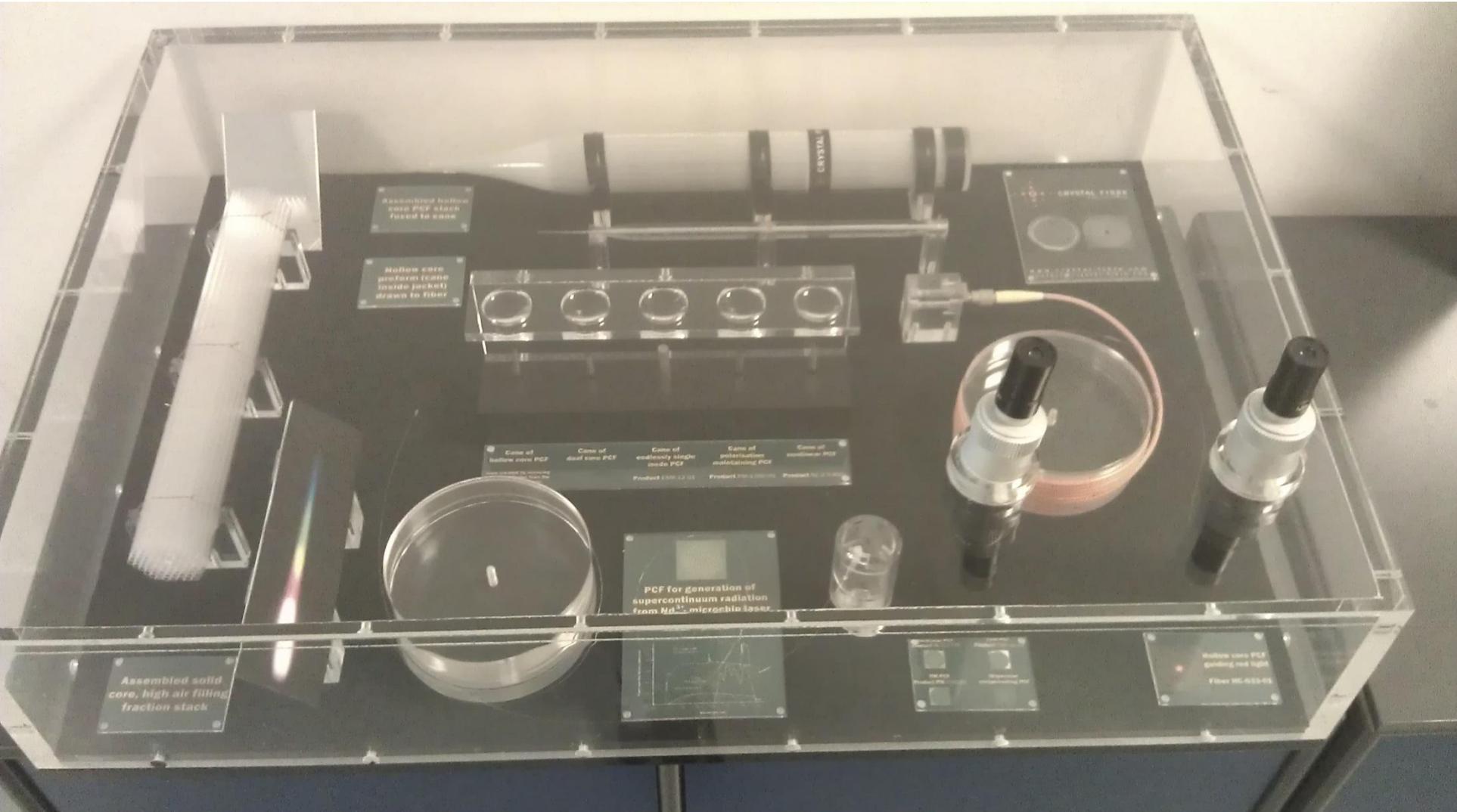
- Sensori e biosensori
- Gas cells
- Trasmissione a lunghezze d'onda "esotiche"



Possibilità di introdurre gas e/o liquidi nel core e massimizzare l'interazione col campo e.m.

Minima frazione di campo e.m. nel vetro, utile per tx fuori dalla finestra di trasparenza

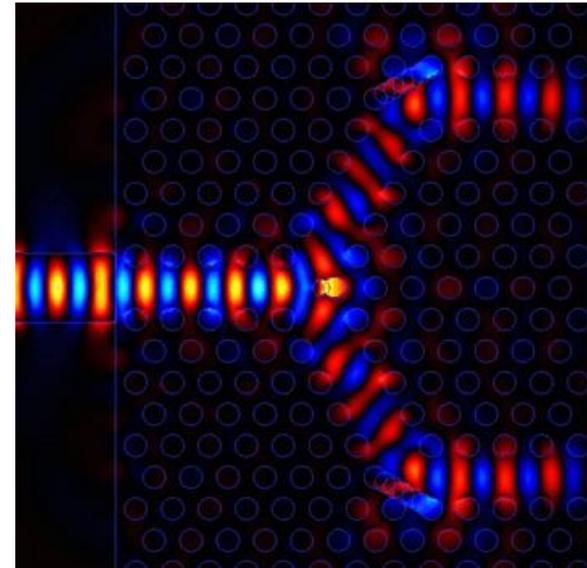
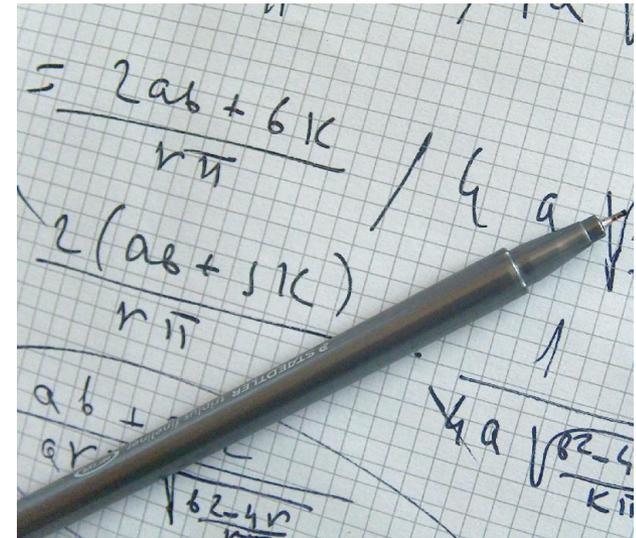




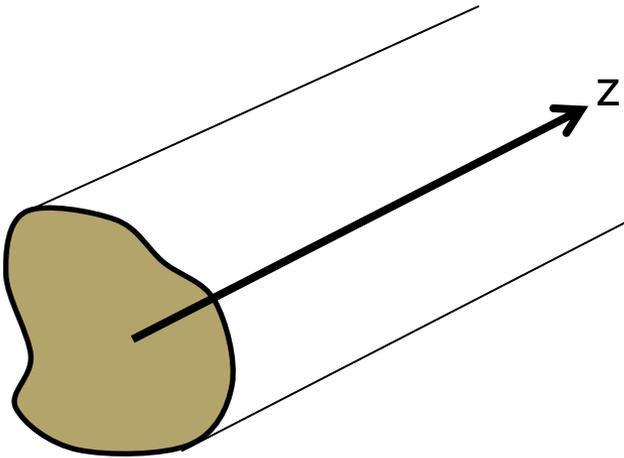


ANALISI E PROGETTO DI PCF

- Teorema di Bloch
- Metodo dell'indice efficace
- Metodo dell'espansione in onde piane
- Metodo delle funzioni ibride ortogonali
- Metodo multipoli
- Metodo della matrice di diffusione
- Metodo delle differenze finite
- **Metodo degli elementi finiti**
- ...



- **Guide rettilinee** (metalliche o dielettriche) si risolvono nell'Hp di **struttura cilindrica**



Struttura cilindrica:

caratteristiche geometriche ed elettromagnetiche invarianti lungo un asse

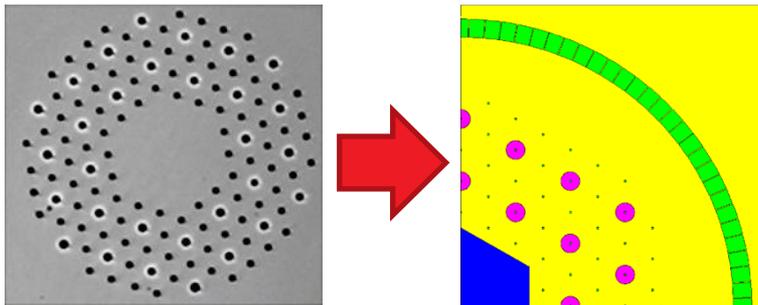
Modi:

Propagazione lungo z

$$\begin{aligned}\bar{e}(x, y, z) &= \bar{E}(x, y) \cdot e^{-\gamma(\omega)z} \\ \bar{h}(x, y, z) &= \bar{H}(x, y) \cdot e^{-\gamma(\omega)z}\end{aligned}$$

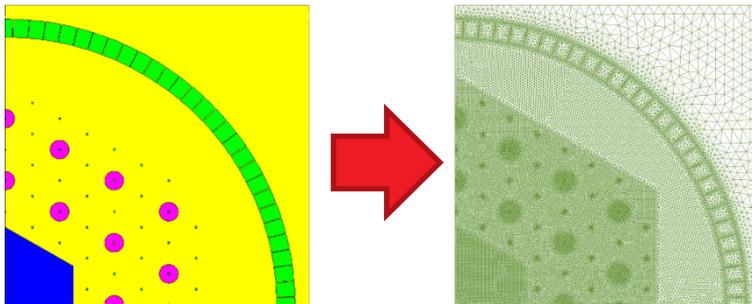
Vettori campo elettrico e magnetico

Distribuzione sul piano trasverso



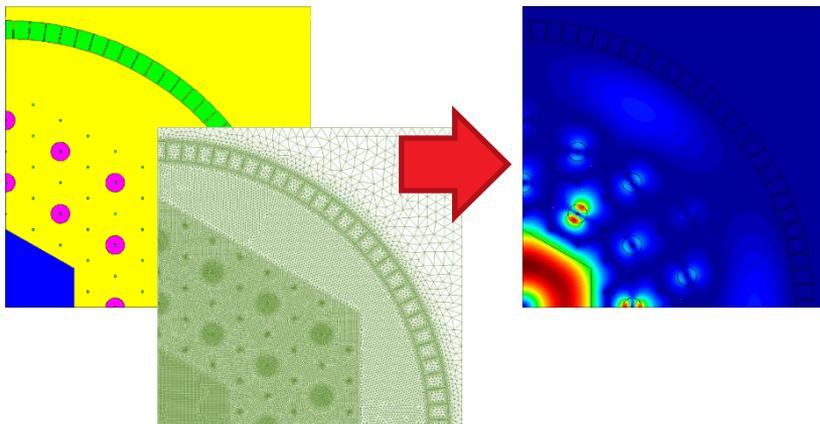
MODELLO DEL PROBLEMA

definizione delle proprietà
geometriche e fisiche

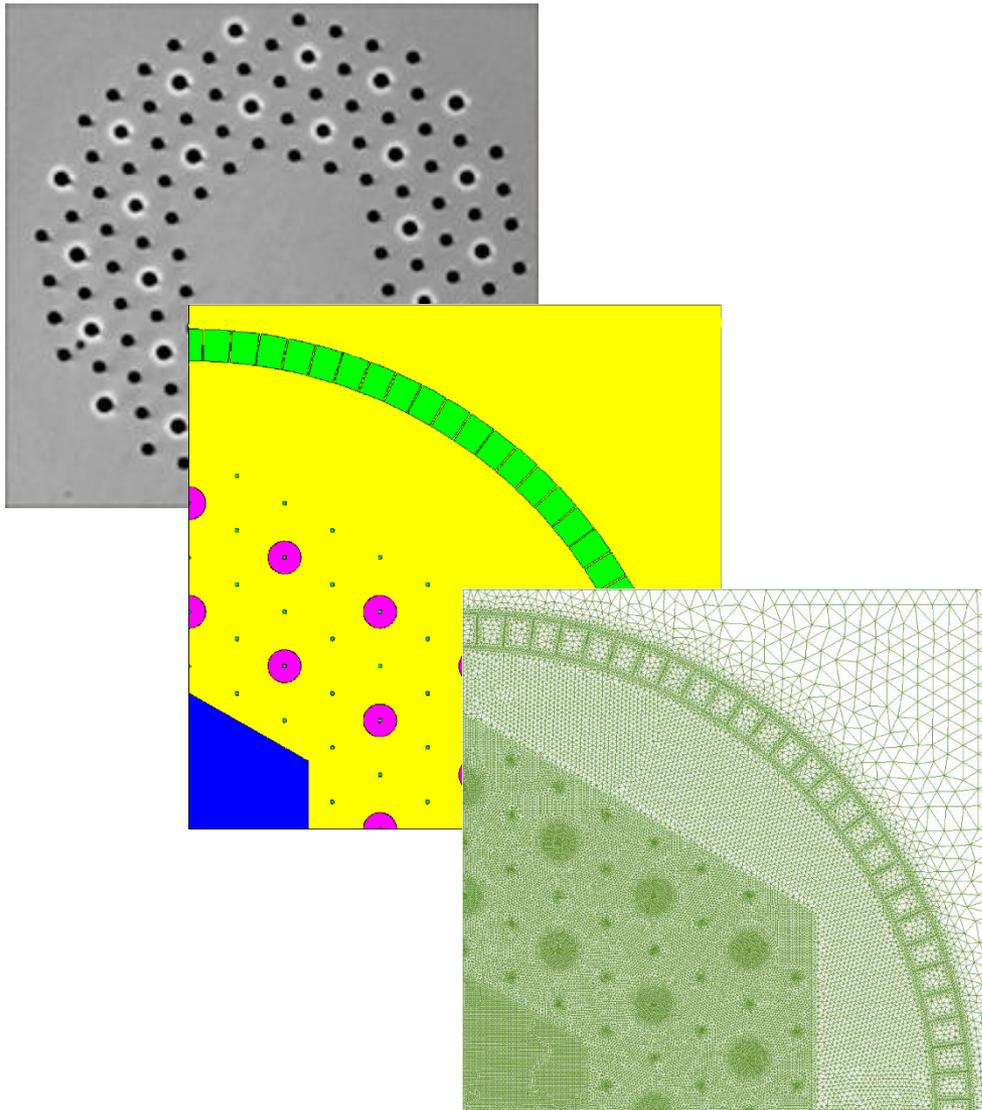


DISCRETIZZAZIONE

creazione della mesh



SOLUZIONE E POST-PROCESSING



Basato sulla discretizzazione del dominio di calcolo in un reticolo di elementi discreti (mesh)

Le equazioni di Maxwell sono risolte come un sistema di equazioni lineari

Metodo generale e numericamente stabile

Precisione della soluzione dipendente dalla qualità della mesh

Curl-curl equation: $\nabla \times (\overline{\epsilon_r})^{-1} \nabla \times \overline{h} - k_0^2 \overline{\mu_r} \overline{h} = 0$ wave number $2\pi/\lambda$
dielectric permittivity magnetic permeability

Modal solution magnetic field: $\overline{h} = \overline{H} e^{-\gamma z}$ field distribution on the cross-section

Complex propagation constant: $\gamma = \alpha + jk_0 n_{eff}$
attenuation constant effective index

Variational finite element procedure

$$\text{Algebraic problem: } \left([A] - \left(\frac{\gamma}{k_0} \right)^2 [B] \right) \{H\} = 0$$

$\{H\}$: discretized magnetic field vector distribution

$[A]$, $[B]$: sparse and symmetric matrices \Rightarrow efficient resolution

Solutore modale vettoriale



Distribuzione di campo dei
modi guidati

Distribuzione di intensità

Area efficace

Coefficiente non lineare

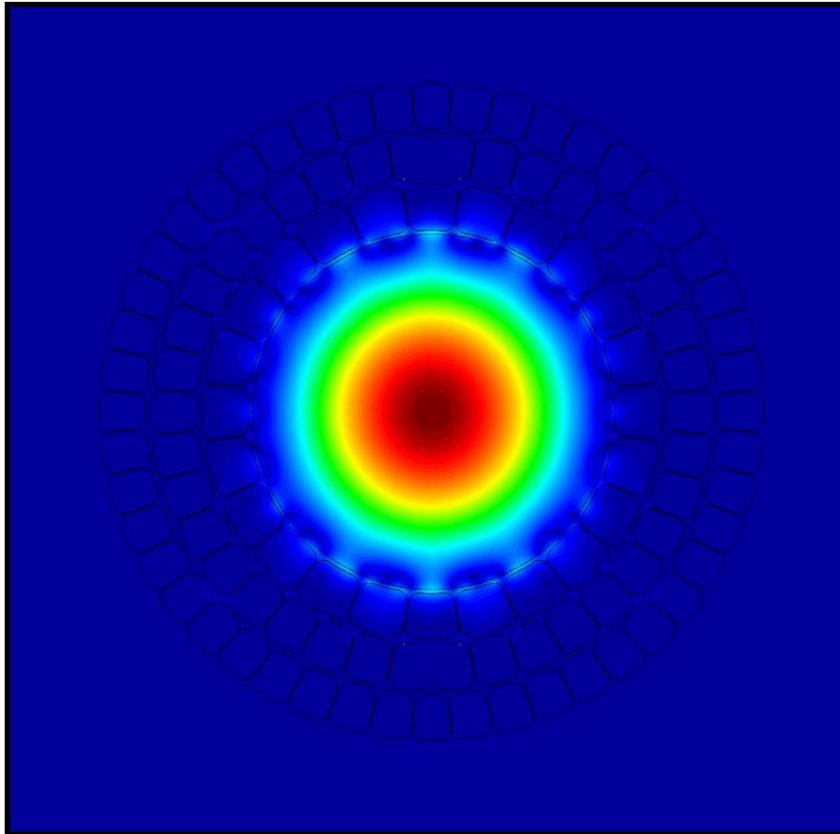
Coeff. di guadagno Raman

Cost. di propagazione complessa



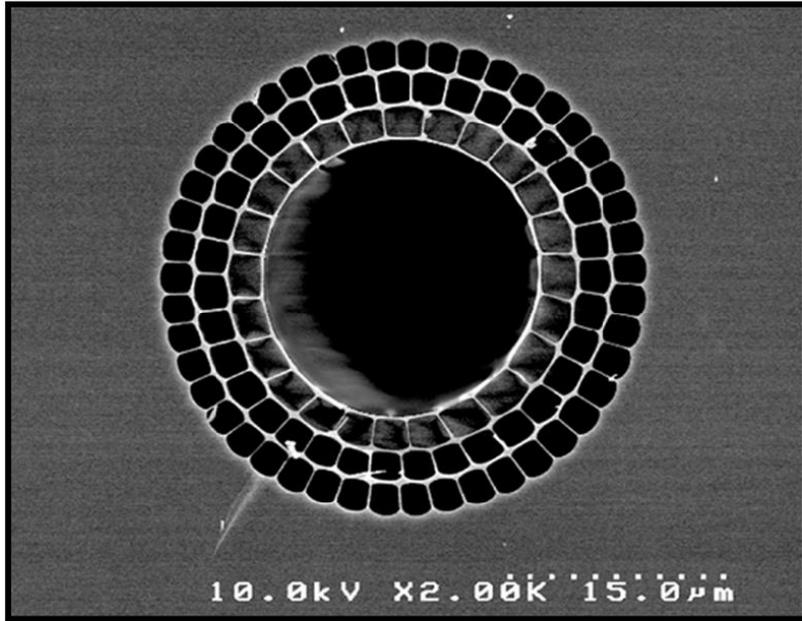
Indice efficace

Cost. di attenuazione

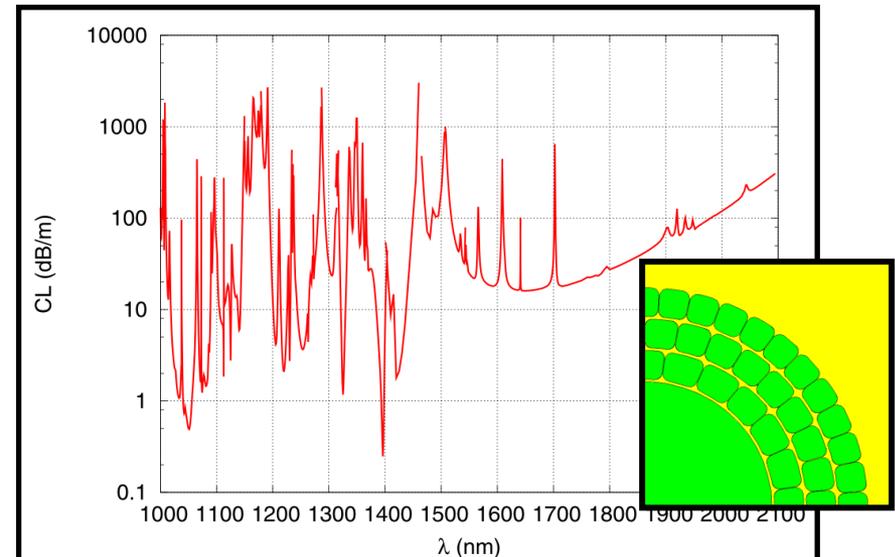
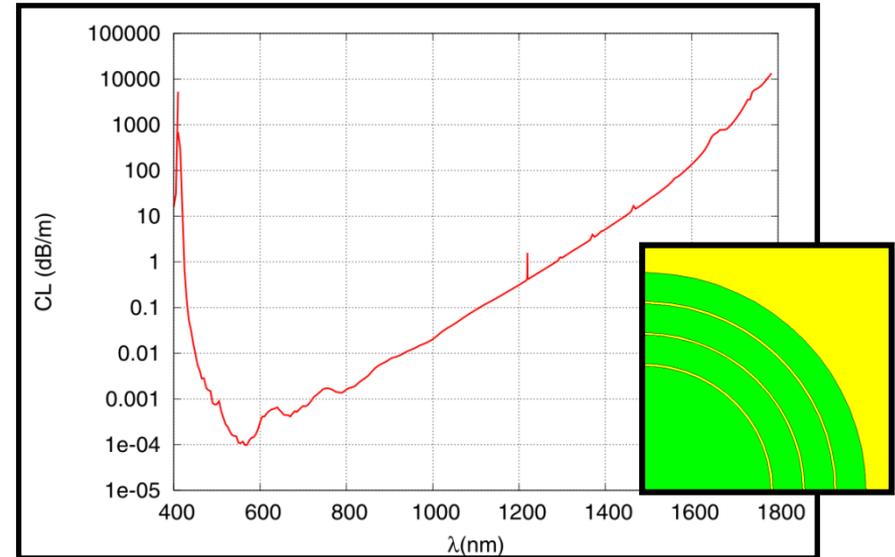


Curva di dispersione $n_{\text{eff}}(\lambda)$
Parametro di dispersione D

Perdite di
confinamento

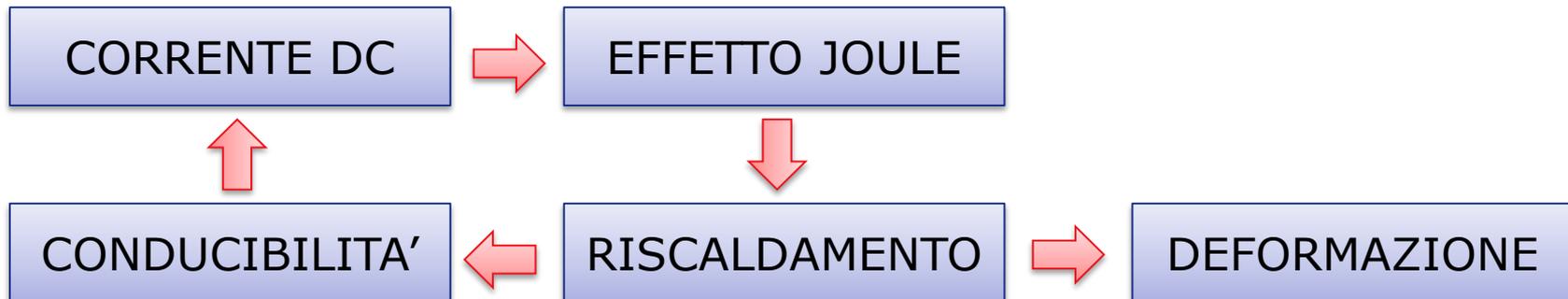
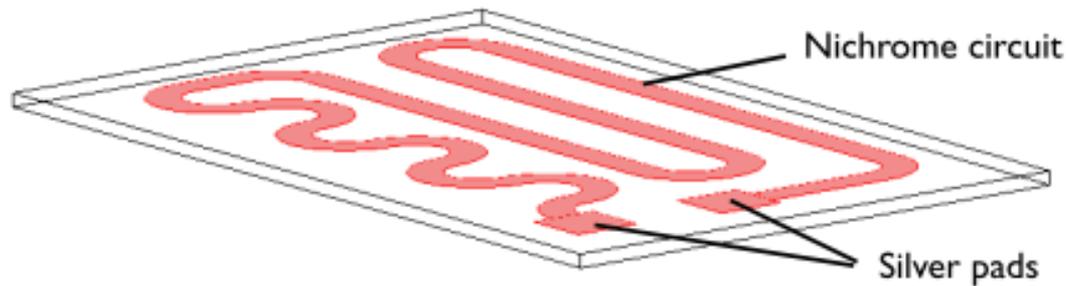


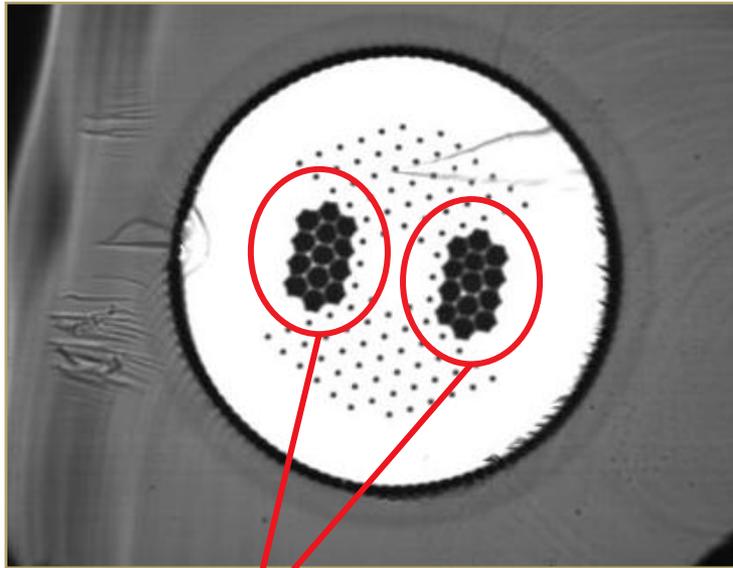
Fondamentale una buona approssimazione della struttura reale!



- Il metodo degli elementi finiti può essere applicato allo studio contemporaneo o sequenziale di fenomeni fisici diversi

es: serpentina elettrica

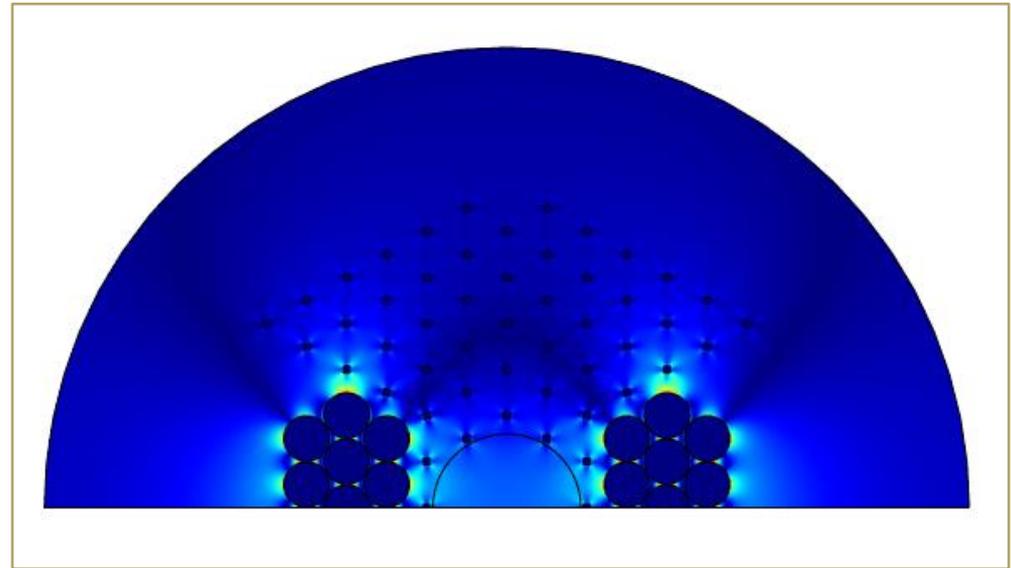




Silice drogata con boro

$$\bar{\bar{n}} = \begin{bmatrix} n_{xx} & 0 & 0 \\ 0 & n_{yy} & 0 \\ 0 & 0 & n_{zz} \end{bmatrix}$$

Proprietà dipendenti dalla polarizzazione del campo



Materiali con diverso coefficiente di espansione termica



Campo di tensione generato durante la filatura



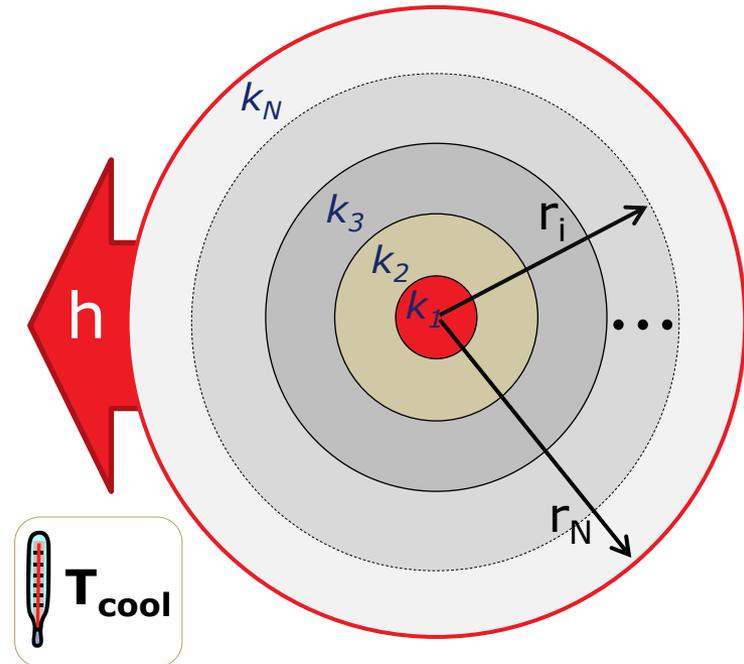
Variazione anisotropica dell'indice di rifrazione

Effetto Thermo-ottico

- Generazione di **calore**
- Gradiente di temperatura
- Variazione di indice di rifrazione
 $\Delta n = \beta \cdot \Delta T$, $\beta = 1.16 \times 10^{-5} \text{ 1/K}$ nella silice



Cambiamento delle proprietà di guidaggio

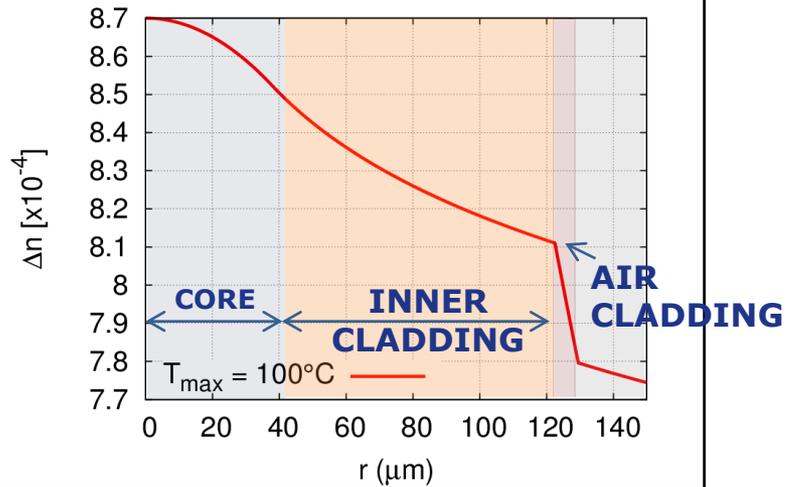


Modello termico

- Sezione della PCF: anelli concentrici, caratterizzati da k_i, r_i
- Densità di calore generato nel core Q_0
- Convezione alla superficie esterna, h

- k_1, \dots, k_N : conducibilità termica (silice: 1.38 W/(m K))
- r_1, \dots, r_N : raggio esterno
- T_{cool} : temperatura esterna

Effetti termici in DC-PCF a 19 celle



- Rod 19-celle, cladding con 4 ring di fori d'aria
- Per $T_{\max} = 100^{\circ}\text{C}$, $\Delta n_{\max} = 8.7 \times 10^{-4}$
- Modi guidati più confinati
- Calo di A_{eff} del 12.6% circa a 1300 nm

